QÜESTIIÓ, Vol 15, 2. pp. 177-197, 1991

# A MODEL AND AN ALGORITHM FOR THE GROUPING PROBLEM IN TELECOM NETWORKS

**F. GARCÍA, K. D. HACKBARTH**

Telefónica I+D, Madrid

*This paper deals with the so called grouping problem which arises in telecommunication network optimization when transmission equipment with a certain hierarchy is used.*

*It is shown how this problem is embedded in the complete scheme of network optimization and a general combinatorial model is developed to derive afterwards a solution procedure.*

*The implementation of this procedure forms part of a software tool for the complete problem of transmission network planning. Some results deriving from practical examples and further extensions are indicated.*

**Keywords:** Telecom Network, Transmission Network, Network Optimization, Grouping Problem.

## 1. INTRODUCTION

A telecom network can be viewed as a set of exchanges with a certain number of links connecting them to transmit information (voice, data, etc) corresponding to a quantity of demands for communication between a set of subscribers.

–Article rebut l'octubre de 1990.

Each subscriber is physically connected to a switching centre (or switching node) that is able to establish the adequate connection for each demand with another subscriber. This connection can be made directly by a circuit between the respective centres or by a set of circuits forming a logical path, having intermediate switching centres (see fig. 1.a.).

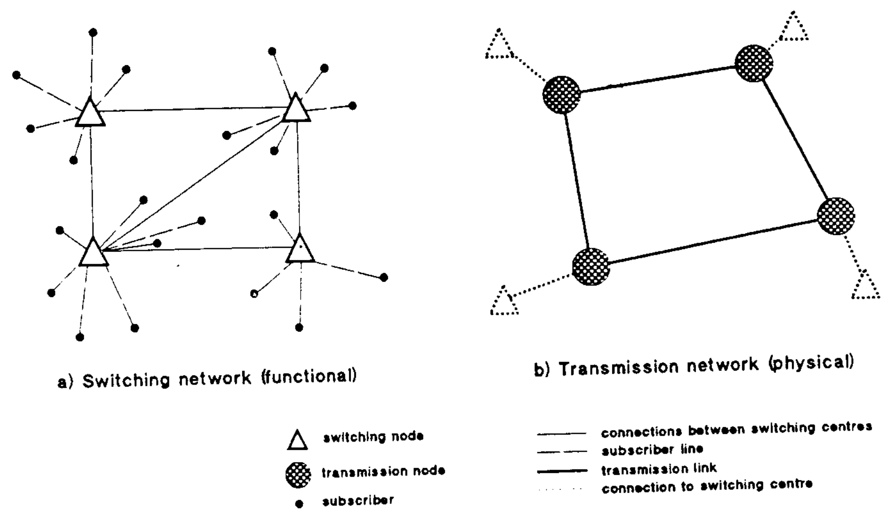


**Figure 1.** Example of a telecommunication network.

A circuit is characterized by a certain value associated to the electrical signal that contains the information. For instance, a telephone call represents a signal of 64 Kbit/s. In modern networks, the digitalization of the equipments has permitted the introduction of many new services such as facsimile, videotext, etc. (see [2]). This forms a functional structure containing a certain number of circuits between pairs of centres.

The number of circuits necessary between two centres depends on the total demand calculated for the subscribers of each centre, which is called telecom traffic. Due to economical reasons, these signals are packed in a single circuit to form a new signal with a higher velocity, saving thus a certain number of circuits. In Europe, the standard is established in a set of 30 signals of 64 Kbit/s that forms a new signal of 2 Mbit/s (***basic signal*** or ***basic circuit.***)

The network composed by these switching centres and the related number of circuits will be called *switching network* or *functional network*.

The previous structure has to be physically realized to transmit the required signals. In most cases, the circuits of 2 Mbit/s do not allow an economical physical realization in form of a proper transmission system (in digital networks, systems carry signals of at least 140 Mbit/s). Therefore, a group of circuits must be concentrated up to the input level of the corresponding systems, connecting a pair of what are called transmission nodes. This concentration must be made following a determined hierarchy of signals. For digital networks, the necessary steps up to the input level or systems are summarized by the next table.

**TABLE I**

Levels of the digital transmission hierarchy

| Level | Bit rate | Nb. of circuits of 2 Mbit/s | Corresponding multiplexer |
|---|---|---|---|
| 1 | 2 Mbit/s | 1 | ———— |
| 2 | 8 Mbit/s | 4 | 2/8 |
| 3 | 34 Mbit/s | 16 | 8/34 |
| 4 | 140 Mbit/s | 64 | 34/140 |

To pass over the intermediate signals (8 and 34 Mbit/s) up to the 140 Mbit/s level a special equipment called *multiplexers*, located in the nodes, has to be used. Each multiplexer has a fixed maximum number of lower level circuits as input and transforms them in one circuit of the next higher level. The complete scheme can be viewed in fig. 2.

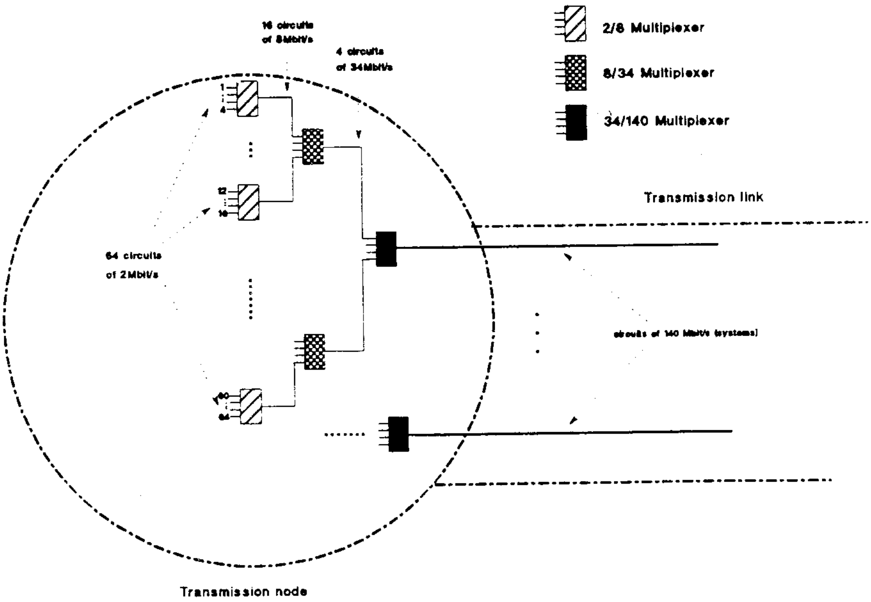


**Figure 2.** Multiplexing structure to transform circuits of 2 Mbit/s up to the 140 Mbit/s level.

The network composed by these transmission nodes and a set of links will be called *transmission network* or *physical network* (see fig. 1.b).

Then, the problem of planning telecom networks is divided into the following two subproblems: Switching Network Optimization (SNO) and Transmission Network Optimization (TNO). The flow between them can be described through the following scheme:

Traffic (number of calls)
↓
SNO
↓
Circuits (number of signals for transmission)
↓
TNO
↓
Optimal configuration of the network

a) SNO deals with the problem of finding the most optimal number of basic circuits of 2 Mbit/s able to carry the fraffic for the whole network. This traffic consists of the predicted average number of calls, calculated from some determined distributions. SNO uses the models arising from Queueing Theory.

b) TNO determines, from the number of circuits calculated by SNO (demand of *basic signals* for some pairs of transmission nodes), the most economical network configuration. This includes: number of systems (equipment to send the information through a transmission medium) for each transmission link and number of multiplexers in each transmission node. The algorithms used in TNO comes from Combinatorial Optimization. The economical importance of TNO es clear, since it takes more than 70% of the total cost of a network.

For practical reasons, the *complete scheme of TNO* is divided into four main subproblems:

- finding physical routes in the network for each demand of basic signals between two transmission nodes, (ROUTING)
- joining basic signals that share a certain subroute to form groups of circuits of a higher level up to 140 Mbit/s. (GROUPING)
- calculating the necessary physical systems to realize the grouped demand. (SYSTEM ASSIGNMENT)
- protecting the systems against failures by means of a stand-by network. (STAND-BY)

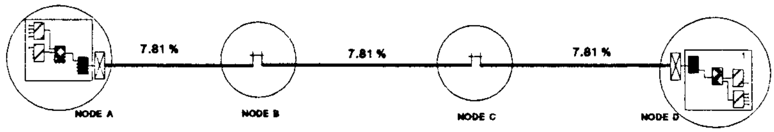


fig.3.1 - A single system for a demand of 5 circuits

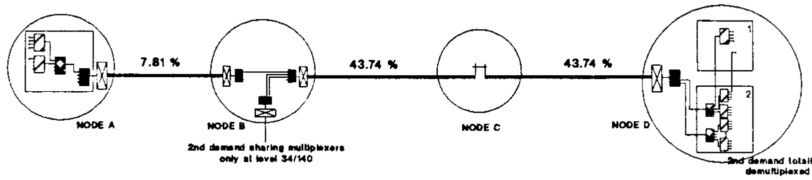


fig.3.2 - Sharing subroute BCD with a 2nd demand of 23 circuits ending at node D

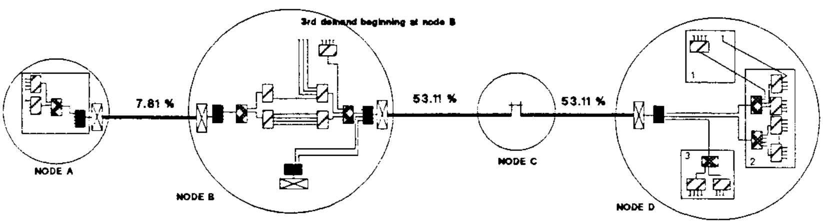


fig.3.3 - Sharing subroute BCD with the 2nd demand and a 3rd one of 7 circuits between nodes B and D

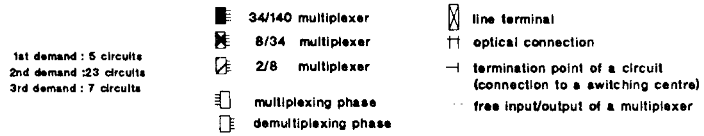


**Figure 3.** Some multiplexing structures from a given demand between two transmission nodes.

The problem of TNO is very complex because all the included tasks are interrelated, especially in the routing and grouping subproblems: grouping starts from the base of a established routing and, conversely, results obtained from grouping may show some edges with a poor utilization degree, being able to solve it by rerouting the circuits of the demands involved over other paths (see [1]).

In long-term planning, [1] both problems will be mostly treated separately because there are no restrictions in the formation of routes, obtaining a target network to be achieved, whereas in medium-term planning the existing capacities and facilities installed have to be taken into account. The type of grouping strategy used will determine a certain utilization degree, depending on the planner's decision leaving a poor fulfilled network because of the expected future demands or because the cost of the systems is worthier than the expensive cost for multiplexer.

The example in fig. 3.1. illustrates these possibilities. Between the nodes A and D there is a demand of 5 basic signals, routed through the links AB-BC-CD without demultiplexing at the intermediate nodes. Suppose there is a second demand of 23 basic circuits between node D and another external node connected to B such that its route has the links BC-CD in common with the one of the first demand. This allows to share the same system instead of installing separate systems per each demand, being only necessary a 34/140 demultiplexing phase in node B (see fig. 3.2.). In the common subroute BCD, the system installed has now a utilization degree of 43.74% instead of 7.81% in the separate case. This concept is extended more in fig. 3.3, where a 2/8 demultiplexing phase has been introduced for the circuits of the 1st demand in order to share equipment with a third demand of 7 circuits between nodes B and D.

Thus, depending on the cost for multiplexers and systems, it will be better either to achieve a good utilization of the systems at the cost or more multiplexers, or more individual systems for the demands, with a saving in the number of multiplexers but a poor utilization degree of the systems.

It is quite interesting to observe that in the literature, there are a lot of references concercing the routing, assignment and stand-by problems (see for instance, [3], [4], [6], and [8]), but not for the grouping problem, where its intrinsic difficulty and dimension has prevented a more extensive treatment. As a preliminary, it can be seen in [1] the influence of the fact of sharing transmission facilities for the rerouting scheme with a dynamic treatment for medium-term

[1] The objectives of Telecommunication Network Planning and the interrelations between SNO and TNO also depend on the time horizon considered. There are usually three types of studies: short-term, medium-term and long-term planning.

planning. A first approach to the grouping problem as a single one can be viewed in [9], where a model and a heuristic for analog telephone networks are developed. In [7], another model and a general solution for grouping in modern digital networks are indicated.

This paper concentrates on the grouping problem, giving a explicit model and a procedure to solve it in a satisfactory way. The approach presented here differs from the rest in that it is based in the concept of a general group, instead of the point of view of the routes calculated for the routing problem, giving thus a more flexible and general treatment, independent of the type of hierarchy used. The implementation procedure forms part of a computer tool named REFORMA that treats the complete TNO problem.

In the next section we will define the main elements that define a transmission network, from a graph theoretical point of view, followed by a set of rules for grouping transformation to higher levels in order to obtain, finally, the general model for the problem.

Afterwards, we will give the algorithm implemented to solve the problem and indicate the results obtained with the software tool REFORMA. Finally, further extensions and some related problems are indicated.

## 2. THE MODEL

### 2.1 Definitions

Let $G(V, E)$ an undirected graph representing the network, consisting of a set $V$ of nodes (transmission nodes) and another set $E$ of edges (transmission links).

Given two nodes $v_s, v_t \in V$, a **route** between $v_s$ and $v_t$ is a chain in $G(V, E)$, not necessarily simple or elementary, with these nodes as its extremes. We will denote it by

$$P_{st} = \{(e_1 \ldots e_m) \quad \text{chain between} \quad v_s, v_t \in V\}$$

with

$$\begin{aligned} m &= |P_{st}| \equiv \quad \text{number of edges in the route} \\ l_{st} &= l(P_{st}) \equiv \quad \text{total length of the route} \end{aligned}$$

In the example outlined in the introduction, it has been pointed out the importance of the constitution of subroutes from a given route for the grouping process. A formalization of this process is expressed by the concept of **route partition** $\mathcal{P}(P_{st})$ generated from a route $P_{st}$

$$\mathcal{P}(P_{st}) = (P_{s_1 t_1} \cup \cdots \cup P_{s_k t_k})/k \leq m,\ s_i, t_i \in V \quad \text{intermediate nodes of } P_{st} \quad \text{with} \quad t_{i-1} = s_i,\ s_1 = v_s \wedge t_p = v_t$$

with $k = |\mathcal{P}(P_{st})| \equiv$ number of subroutes forming the partition.
For $k = 1, \mathcal{P}(P_{st})$ will be called **trivial partition**.

The magnitude of the grouping problem depends on the number of combinations found during the process. The following lemma gives an idea of the maximum size to be treated:

**Lemma 1**

The total number of possible partitions, different from the trivial one, that derives from a certain route $P_{st}$ with $m$ edges is given by the recursive formula

$$C_m = (m-1) + 2\sum_{j=1}^{m-1} C_j \qquad m > 2,$$

being $C_1 = 0$ and $C_2 = 1$.

**Proof**

The first term is the number of possible partitions with two subroutes, constituted by all the pairs of the form $(i, m-i)$ for $i = 1, \ldots, m-1$. For each element of these 2-partitions $C_i + C_{m-i}$ partitions are derived. The sum of these elements gives the above result. ∎

## 2.2 Characterization of a group of signals

The following gives a formal description of the elements to be defined for a general hierarchy of signals with $L$ levels.

For a certain level $l$ of the hierarchy ($l = 1, \ldots, L$) and a corresponding number of circuits $x_{st}^l$ (signals of level $l$ or inputs of the corresponding multiplexer) which carry informations between two transmissions nodes $v_s$ and $v_t$ through a certain route $P_{st}^l$, we define a **group** between $v_s$ and $v_t$ in level $l$ as

$$g_{st}^l \equiv g_{st}^l(x_{st}^l, P_{st}^l).$$

Thus, a group is characterized by the corresponding route, and it has associated a certain number of signals, a value that is able to change in the intermediate stages of the grouping process. In the case that $x^l_{st} = 0$, $g^l_{st}$ is called **empty group**.

Let $n_l$ the maximum number of signals of level $l$ that can be embedded in an upper group of level $l+1$ (capcity of a $l$–level group). Then it holds that

$$x^l_{st} < n_l$$

because otherwise, we can assign $\left[\frac{x^l_{st}}{n_l}\right]$ signals to the next level and work with $x^{*l}_{st} = x^l_{st} - \left[\frac{x^l_{st}}{n_l}\right] \cdot n_l$ signals of level $l$.

### 2.3 Initial grouping

As it was defined before, the routing step of the TNO scheme provides for each demand of basic circuits between two nodes a set of routes over which a portion of the total demand has to be carried.

Once these routes have been calculated, the first step of the grouping process consists in assigning a maximum of $L$ initial groups for each one of these routes.

Following the group definition of 2.2, the routing structure always generates a determined grouping pattern that we will call **initial grouping**.

In this step, the portion of the demand carried by a route is divided according to the structure of the considered hierarchy (e.g. for a digital one the structure presented in the table 1), assigning the possible inputs of the multiplexers that the demand can fulfill by its own. As, after the grouping process, all the circuits must be contained in groups of level $L$, the number of free inputs that the circuits of each route do not fulfill can be used by any group of circuits of other routes being a subroute of the first one, sharing in this way the same multiplexing equipment. An example of the multiplexing structure in one node according to the table 1 is shown in fig. 2.

For instance, a demand of 109 basic circuits in the digital hierarchy of 4 levels (table 1) results in the following distribution: 1 direct circuit of 140 Mbit/s, 2 circuits of 34 Mbit/s (or, equivalently, 2 inputs for one multiplexer of 34/140), 3 of 8 Mbit/s and a single remaining circuit of 2 Mbit/s. To realize this structure, two direct groups of level 4 (140 Mbit/s) are needed, leaving 3 free inputs for one multiplexer of 2/8 Mbit/s and 1 input for the 34/140 Mbit/s one.

Then, the objective of the subsequent grouping process is to improve the utilization of the multiplexers and the systems, joining groups of circuits resulting from the initial grouping. This will be expressed by the concept of group transformation, explained in the next paragraph.

### 2.4 Group transformation

Let $G^l = \{g^l_{st}/s, t \in V\}$ the set of all the groups defined for level $l$, including the ones that are empty. Then, at the beginning of the grouping process, the only non empty groups in $G^l$ are those corresponding to the initial grouping defined in 2.3.

Let a group $g^l_{st}(x^l_{st}, P_{st}) \in G^l$, and $\mathcal{P}(P_{st})$ a route partition induced by $P_{st}$, with $g^l_{s_it_i} \in G^l$ representing the associated groups of each subroute $P_{s_it_i} \in \mathcal{P}(P_{st})$. The following transformation $\pi^l$ joins $g^l_{st}$ with each $g^l_{s_it_i}$:

$$\begin{array}{rccc} \pi^l: & G^l \times G^l & \longrightarrow & (G^l, G^{l+1}, \ldots, G^L) \\ & (g^l_{st}, g^l_{s_it_i}) & \longrightarrow & \pi^l(g^l_{st}, g^l_{s_it_i}) = g^l_{st} \oplus g^l_{s_it_i} \end{array}$$

where

$$\begin{aligned} g^l_{st}(x^l_{st}, P_{st}) \quad & \oplus \quad g^l_{s_it_i}(x^l_{s_it_i}, P_{s_it_i}) = \\ & = \quad g^l_{st}(0, P_{st}) \cup g^l_{s_it_i}\left(\delta(x^l_{s_it_i}) \cdot \operatorname{mod}(x^l_{s_it_i} + x^l_{st}, n_1); P_{s_it_i}\right) \cup \\ (1) \qquad & \cup \quad \bigcup_{j=l+1}^{L} g^j_{s_it_i}\left(\delta\left(\operatorname{mod}(\hat{x}^j_{s_it_i}, n_j)\right) \cdot \hat{x}^j_{s_it_i}; P_{s_it_i}\right) \end{aligned}$$

$$\forall i = 1, \ldots, k$$
$$\forall l = 1, \ldots, L-1$$

$$(2) \qquad \text{being} \quad \hat{x}^j_{s_it_i} = x^j_{s_it_i} + \lceil (x^{j-1}_{s_it_i} + x^{j-1}_{st})/n_{j-1} \rceil + 1$$

(here $\lceil r \rceil$ denotes the smallest integer $\geq r$ ($r \in \mathbf{R}$)), and $\delta(x) = \left[ \begin{array}{ll} 0 & \text{if} \quad x = 0 \\ 1 & \text{if} \quad x > 0 \end{array} \right.$

The formula given by (1) is also valid for the special cases when $P_{s_it_i} \equiv$ empty partition, considering $x^j_{s_it_i} = 0 \ \forall i, j$, or $P_{st} \equiv$ trivial partition, considering a single partition $P_{s_1t_1} \equiv P_{st}$($\equiv$ in the sense that $v_{s_1} = v_s$ and $v_{t_1} = v_t$) associated to an emtpy group $g^l_{s_1t_1}(0, P_{s_1t_1})$.

By definition, each $L-$level group leads to a transmission system with a certain degree of utilization depending on the resulting value of $x^L_{st}$. For reasons of cost saving, it is often worthwhile to apply the former group transformation $\pi$ at level $L$, obtaining thus a final set of groups with a higher value of their corresponding $x^L_{st}$.

The formula given in (1) is also valid for $l = L$, but it leads to a great simplification in its expression because of the empty union formed by the second term of the formula. This results in

$$\begin{array}{llll} \pi^L : & G^L \times G^L & \longrightarrow & G^L \\ & (g^L_{st}, g^L_{s_it_i}) & \longrightarrow & \pi^L(g^l_{st}, g^L_{s_it_i}) = g^L_{st} \oplus g^L_{s_it_i} \end{array}$$

$$\begin{aligned} g^L_{st}(x^L_{st}, P_{st}) \;&\oplus\; g^L_{s_it_i}(x^L_{s_it_i}, P_{s_it_i}) = \\ &= \; g^L_{s_it_i}\;(\delta(s^L_{s_it_i}) \cdot \operatorname{mod}(x^L_{s_it_i} + x^L_{st},\, n_L); P_{s_it_i}) \end{aligned} \tag{3}$$

$$\forall i = 1, \ldots, k$$

with $\delta$ the same function as in (2).

Observe that, after having grouped one $g^l_{st}$ with only a single element $g^l_{s_it_i}$ of the partition, the set $G^l$ does not change in the quantity of its elements, except in the number of signals associated to the groups involved in the transformation:

$$\begin{array}{lll} g^l_{st}(x^l_{st}, P_{st}) & \text{changes to} & g^l_{st}(0, P_{st}) \\ g^l_{s_it_i}(x^l_{s_it_i}, P_{s_it_i}) & \text{leads to} & g^l_{s_it_i}(\delta(x_{s_it_i}) \cdot \operatorname{mod}(x^l_{st} + x^l_{s_it_i}, n_l), P_{s_it-i}) \end{array}$$

$$\forall g^l_{s_it_i} / P_{s_it_i} \in \mathcal{P}(P_{st})$$

For the rest of the levels up to $L$, the number of signals in each group depends on the value of the module of the hierarchy, as it is shown in formulae (1)–(3). The reduction in the number of non empty groups of $G^l$ is expressed by the following lemma, where $|G^l| \equiv$ number of $g^l_{st}$ with $x^l_{st} > 0$

**Lemma 2**

Let $g^l_{st} \in G^l$, $l \leq L$ and $\mathcal{P}(P_{st})$ an arbitrary route partition with $|\mathcal{P}(P_{st})| = k$. Then, after having applied each group transformation $g^l_{st} \oplus g^l_{s_it_i}$, it holds that

$$\begin{array}{ll} |G^l| - k \leq |G^l_\pi| \leq |G^l| - 1 & \\ |G^{l+i}| - k \leq |G^{l+i}_\pi| \leq |G^{l+i}| + k & \quad 1 \leq i \leq L-1 \end{array}$$

denoting by $G^l_\pi$ the set $G^l$ after the transformation defined above.

**Proof**

Immediate by induction.

■

As a consequence of this lemma, $|G^l| = 0$ after having grouped all the elements in $G^l$ with a number of signals greater than zero.

The next lemma shows the consequence of the group transformation in level $l$ with respect to the lower and upper ones.

**Lemma 3**

A group transformation in a level $l$ changes at least one value of the number of circuits $x_{s_it_i}^{l+k}$ for a certain group $g_{s_it_i}^{l+k}$ of higher leven than $l$, but does not change any groups in the lower levels.

**Proof**

It is evident from the application of lemma 2 and the definition of group transformation.

■

### 2.5 Special group transformations

For some types of route partitions, the resulting group transformation leads to particular cases, very usual in practical realization of telecom networks.

Given a group $g_{st}^l(x_{st}^l, P_{st}) \in G^l$ with $m$ edges forming the associated route $P_{st}$, then

a) A route partition $\mathcal{P}(P_{st})$ such that $|\mathcal{P}(P_{st})| = m$, leads to subgroups $g_{s_it_i}^l$ with associated subroutes having $|P_{s_it_i}| = 1$. This transformation is called **edge grouping**.

b) On the contrary, the case of grouping $g_{st}^l$ with its trivial partition leads to an **end-to-end grouping.** This is equivalent to a transformation of $g_{st}^l$ with an empty group.

c) Finally, we will call **basic route grouping** the case when the groups $g_{s_it_i}^l$ formed after each group transformation are the same for all the levels $1, \ldots, L$.

### 2.6 Cost of a group transformation

The economical realization of a set of transformations is composed by two types of costs, depending on the level:

a) Multiplexing cost ($M_l$), for each level $l$ up to $L$.
As it was defined before (see formulae (1)–(3) and fig. 2), each transformation of any number of circuits to the next level ($x_{st}^l \longrightarrow \hat{x}_{st}^{l+1}$, quantity that depends on the module $n_l$) represents a number of $\hat{x}_{st}^{l+1}$ multiplexers of the corresponding type at both ends of the route.

b) Transmission cost for level $L$.
We first have the cost of the systems, that is referred to the necessary equipment to send the information at a determined velocity. [2] Each $L$–level group corresponds to a transmission system, being necessary one equipment at each end of the associated route. Thus, the total cost is formed by the number of non empty $L$–level groups not being transformed in subgroups, multiplied by the cost per unit of equipment ($S_L$) for each extreme (source and sink transmission nodes), that is

$$C_S = 2|G^L|S_L$$

Finally, we have to consider the cost of the physical realization of the route associated to each $L$–level group, $f(P_{st})$, that is the cost per length of the physical transmission medium associated to each link of $P_{st}$ (cost of the optical fibre installed). Obviously, for two routes $P_{st}$ and $\overline{P}_{st}$ with $l(P_{st}) \geq l(\overline{P}_{st})$, it holds that $f(P_{st}) \geq f(\overline{p}_{st})$.

In most applications, $f(P_{st})$ is usually of the form:

$$f(P_{st}) = l_{st}(A_T + B_T x_{st}^L)$$

where $A_T$ is a fixed cost that always appears when a cable or fibre is installed, $B_T$ is the cost per unit of length of the medium considered and $x_{st}^L$ is the number of systems of the route to be realized.

Thus, for a given transformation $\pi^l \equiv \pi^l(g_{st}^l, g_{s_i t_i}^l)$, the associated costs for each level depends on the formed groups according to the expressions (1)–(3) given before and will be summarized in the following formulae:

a) Multiplexing cost (for one transformation within one level from 1 to $L$)

$$C_M(\pi^l) = 2M_l\delta(x_{s_i t_i}^l) + \sum_{j=l+1}^{L} 2M_j\delta\left(\mathrm{mod}(\hat{x}_{s_i t_i}^j, n_j)\right)$$

with $\hat{x}_{s_i t_i}^j$ and $\delta$ defined as in (2).

[2] The last level for groups ($L$) is the first one for the hierarchy of systems that is used in the system assignment step of TNO and is based in the velocity (140 Mbit/s, 565 Mbit/s and 2.4 Gbit/s in actual digital networks). However, in the grouping step, only level $L$ is necessary.

b) Transmission cost (for each group of level $L$)

$$C_T(g^L_{s_i t_i}) = 2S_L + f(P_{s_i t_i})$$

### 2.7 Optimal cost problem

The grouping problem starts from the set of routes calculated during the routing step for each demand. The initial grouping gives the separate configuration of the groups for each demand through the respective levels. Since all the intermediate level groups must be multiplexed up to the highest level $L$, and taking into account the inherent costs for the multiplexers and systems, it seems clear that the group structure must be configured in order to minimize these costs, searching for the best way of joining groups from different demands. In practice, this structure has to be achieved level by level, beginning with the lowest one because of the result from lemma 3.

The function $\pi$ defined in 2.4 permits the transformation between a prescribed level up to the next one, reducing the number of non empty groups for each intermediate level, as it was demonstrated by lemma 2. Besides, it modifies the composition of the groups by joining circuits from different demands. In this way, $\pi$ leads to a common multiplexing structure, reducing the cost. Thus, $\pi$ seems a good candidate to be used for achieving the optimal cost configuration.

Once the grouping function $\pi$ has been defined, given an initial set of groups $G^l$ for each level $l = 1, \ldots, L$, the problem of grouping consists in finding the set of group transformations leading to level $L$ that minimizes the sum of the costs defined before. This results in the following objective function to be optimized

$$(4) \qquad \min_{\pi} \left( \sum_{\{(\pi^1 \ldots \pi^L)/|G^l|=\emptyset, \forall l=1,\ldots,L-1\}} C_M(\pi^l) + \sum_{g^L_{st} \in G^L} C_T(g^L_{st}) \right)$$

where the first sum is referred to the set of succesive transformations needed to achieve $|G^l| = 0$ for each level up to $L - 1$.

## 3. ALGORITHM

Before starting with the description of the algorithm, let us consider some remarks. The first one comes from the definition of group transformation, which requires that one of the routes is a subroute of the other one. This leads to the question of starting the procedure with the routes having the smallest number of edges, trying to find a longer route that includes this former one or, vice versa, to begin with the routes with more edges looking for their best partition.

The fact that the cost of the uppest level groups depends on the length of the associated route, as it was seen in 2.6, implies that it is better to proceed downwards, to ensure the optimal grouping for the longest, and so more expensive, routes.

For any group $g^l_{st}$, let us define $U_{st}$ as the set of the feasible partitions whose elements can be grouped with $g^l_{st}$. The choice of the optimal one is also based on economical reasons, depending of the number of circuits of each $g^l_{s_i t_i}$. A smaller of circuits would produce low fulfilled upper groups, resulting as a consequence low fulfilled systems.

Let us suppose that the routing part of the TNO problem and the initial grouping have been already done. Once all the routes are sorted in decreasing order of number of edges, let define $l_{\max}$ as the maximum number of edges among all the routes. Then, the general algorithm for grouping is the following:

```
DO for all levels l = 1, ..., L
    DO for all number of edges n = l_max, 1
      DO for all g^l_st with |P_st| = n
         U_st <-- ∅
         DO for all g^l_{s_i t_i} with |P_{s_i t_i}| = k < n and P_{s_i t_i} ∈ 𝒫(P_st)
            U_st <-- U_st ∪ {g^l_{s_i t_i}}
         END DO
      IF U_st not ∅ THEN
            Let g^{l*}_{s_i t_i} the optimal group in U_st
            Form g^l_st ⊕ g^{l*}_{s_i t_i}
            Repeat for each g^{l*}_{s_i t_i} of the optimal partition
      ELSE
         Form g^l_st ⊕ g^{l*}_st(0, P_st)
         END IF
      END DO
    END DO
END DO
```

The algorithm works in each level separately treating each route of a certain number of edges with a non empty associated group. For each one of these routes, all the possible route partitions with associated non empty groups are searched, applying the corresponding group transformation of there exists any, or resulting in an end-to-end grouping otherwise. This is repeated until the succesive group transformations lead to a set of groups for the level with all its elements being empty groups.

The process consists of a single iteration in which a local minimum is found from a sequence of local minima obtained in each step of the loop. Its convergence is guaranteed because the number of possible subgroups is bounded (lemma 1).

This algorithm includes as particular cases the special types of grouping defined in 2.5.

## 4. IMPLEMENTATION AND USE

The grouping algorithm has been implemented in FORTRAN/77, forming part of a complete program for calculating the grouping process of TNO. Besides, all the special strategies defined before in 2.5 have been implemented separately and can be selected through a menu. As it was shown, these strategies appears when the group transformations are forced to be done only with groups associated to special types of route partitions.

The size of the grouping is given by the total number of groups calculated during the process. Because of the size resulting from the information associated to each group, taking also into account the formula of lemma 1 for the set of total routes of any network, the characteristics of each group have to be stored in a direct access file. There, each address corresponds to an unique group, being classified by levels. Each record contains the succesive addresses of the groups formed up to level $L$, obtaining a tree structure easy to follow.

The grouping program forms part of a complete tool for the task of TNO called REFORMA. This tool is being succesfully used by the planning departments of Telefónica to configure urban and long-distance networks, starting from a given topology and a set of demands in circuits of 2 Mbit/s between some pairs of transmission nodes.

The main difference between urban an long-distance networks consists in the additional calculation of the equipment for regenerating the signal approximately every 100 km. On the other hand, urban networks can also be hierarchical, in the sense that there exists two types of transmission nodes: from the lower hierarchy (primary nodes) and from the upper one (secondary nodes), assigning a certain number of secondary nodes (generally two) to each primary node. This structure adds a new constraint to the formation of routes, because each one having as one extreme a lower level node has to pass over the associated upper level nodes. These subroutes are fixed for all the demands having this node as one extreme, being thus a candidate for basic route strategies. (see [5] for more details). For long-distance networks, it is not worthy considering hierarchy because of the signal regeneration.

The grouping module, that comprises the initial process, the general algorithm and the process of system assignment, is very fast. The table in fig. 4 gives an idea of the types of execution that can be made with the tool and the resulting runtimes. They have been obtained with a PC-386. The general grouping algorithm has been implemented for urban or local networks, being the version for long-distance ones under study because of the additional problem of the intermediate regenerators.

| Types of examples. Characteristics | | Urban network | Random network | Long-distance network | Random network |
|---|---|---|---|---|---|
| Number of nodes | | 12 | 25 | 67 | 80 |
| Number of edges | | 18 | 36 | 96 | 115 |
| Number of demands | | 56 | 140 | 615 | 398 |
| Number of routes | | 112 | 280 | 1230 | 796 |
| MULTIPATH ROUTING + END-TO-END GROUPING | Nb. of Initial groups | 134 | 477 | 1480 | 114 |
| | Total nb. of groups | 359 | 763 | 4444 | 2075 |
| | Execution time | 14s | 36s | 2m 53s | 3m 16s |
| MULTIPATH ROUTING + EDGE GROUPING | Nb. of Initial groups | 134 | 477 | 1480 | 1162 |
| | Total nb. of groups | 185 | 562 | 1724 | 562 |
| | Execution time | 18s | 1m 02s | 3m 48s | 1m 02s |
| MULTIPATH HIERARCHICAL ROUTING + BASIC ROUTES GROUPING | Nb. of Initial groups | 134 | 481 | (*) | 1162 |
| | Total nb. of groups | 211 | 618 | | 1758 |
| | Execution time | 19s | 19s | | 3m 30s |
| MULTIPATH ROUTING + FREE GROUPING | Nb. of Initial groups | 134 | 477 | | 1144 |
| | Total nb. of groups | 218 | 709 | | 1976 |
| | Execution time | 1m 27s | 10m 08s | | 1h 43m 13s |

(*) Not yet available because of the characteristics of the long-distance network.

**Figure 4.** Characteristics of the execution of the grouping process for several types of networks.

## 5. RESULTS

Fig. 5b) shows some results extracted from the grouping module of REFORMA applied to the urban network drawn in fig. 5a), corresponding to a medium size Spanish city.

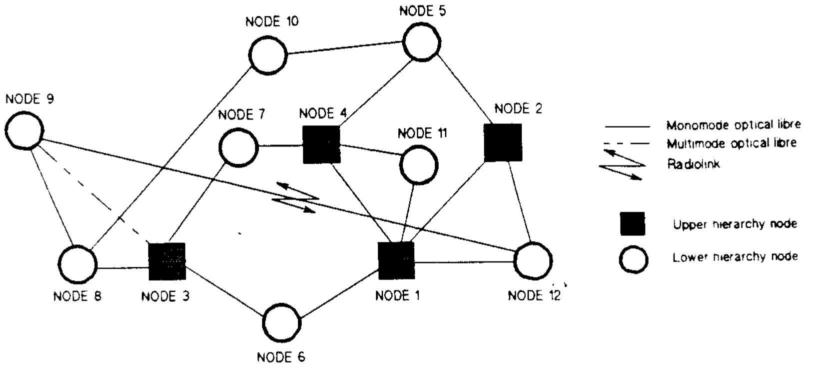


**Figure 5a.** Example of the urban network of a medium size Spanish city.

| ROUTING STRATEGY | GROUPING STRATEGY | NUMBER OF MULTIPLEXERS 2/8 | 8/34 | 34/140 | NUMBER OF SYSTEMS | UTILIZATION DEGREE | TOTAL COST (Millions of pts) |
|---|---|---|---|---|---|---|---|
| Multipath | end-to-end | 94 | 176 | 224 | 112 | 10.06% | 1818.58 |
| | edge | 120 | 154 | 94 | 39 | 65.30% | 870.74 |
| | free | 78 | 118 | 84 | 41 | 54.51% | 836.74 |
| Multipath hierarchical | end-to-end | 94 | 176 | 224 | 112 | 9.62% | 1830.12 |
| | edge | 130 | 162 | 98 | 41 | 67.84% | 903.18 |
| | basic routes | 98 | 120 | 80 | 40 | 52.92% | 844.06 |
| | free | 84 | 112 | 80 | 39 | 61.38% | 814.26 |

**Figure 5b.** Results obtained from the network of fig. 5a.

The end-to-end grouping leads to the minimum number of multiplexers in intermediate nodes (none), but with a poor utilization of the systems (each initial group becomes a single system without sharing facilities with any other one) whereas, on the contrary, edge grouping gives the maximum utilization degree al the cost of the maximum number of multiplexers installed in every node in each group.

The basic route grouping is used for 2-level hierarchical networks. The nodes are divided into primary and secondary ones. Each primary node is assigned to two secondary nodes and all the routes starting or ending in this primary node hast to pass over the secondary ones, being able to group all the demand of these routes.

Finally, the unconstrained grouping or free grouping is the optimal one but, in practice, is very complicated to be achieved because it implies a difficult network maintenance and management of the meshed structure of groups that can result. Anyway, it allows cost comparisons with any constrained grouping strategy, such as basic or edge grouping, to evaluate the deviation from the optimal strategy.

## 6. FURTHER EXTENSIONS

A second stage or REFORMA is actually being developed in Telefónica I+D, including: a new tool for medium-term planning, which leads to the consideration of the existing capacities in the network (there exists a limit in the utilization degree of the systems) and a closer interrelation between the routing and grouping problems; a new module for optimizing the network topology, instead of being an input data, and new extensions for the existing tool, adding new strategies and advanced equipment (for example, direct multiplexers from 2 to 140 Mbit/s) and treating demands of circuits for different services having other velocities than 2 Mbit/s.

Finally, the tool is intended for studying several scenarios of circuit demands for the Broadband Integrated Service Digital Network (B-ISDN) that includes new services such as high definition TV, videoconference, high velocity data, etc, services that require a flexible network with new strategies and equipment.

## 7. REFERENCES


[1] **Doverspike, R.D.** (1986). "Multiplex bundler algorithms for the network planning system". Third International Network Planning Symposium, Innisbrook.

[2] **Frank, H., Frisch, I.T.** (1971). "Communication, transmission and transportation networks". Addison Wesley.

[3] **Hackbarth, K.D.** (1984). "Implementation of algorithms for telecommunication network planning". *Proceedings on Graphtheoretic Concepts in Computer Science,* Linz.

[4] **Hackbarth, K.D.** (1989). "Economical routing strategies on the ISDN with regard to network reliability". *Proceedings of EUROTELECOM,* Madrid.

[5] **Hackbarth, K.D., García, F.** (1990). "REFORMA - a tool based on PC for telecom network optimization". *Proceedings of EUROTELECOM,* Madrid.

[6] **Hackbarth, K.D., Kopp, P.** (1980). "A method for an optimal increase of capacity in the planning of telecommunication transmission networks". *Telecommunication network planning,* París.

[7] **Kopp, P.** (1989). "On solving the grouping problem in the planning of telecommunication transmission networks". Fourth International Network Planning Symposium, Mallorca.

[8] **Minoux, M.** (1974). "Recherche de la configuration optimale d'un réseau de télécommunications avec fonctions de coût concaves". (two parts). *Ann. Telecommunic.,* 29, n. 1-2, 3–4.

[9] **Peyrade, M.** (1977). "La constitution optimale des groupes de niveau supérieur dans le réseau national. Formulation du problème et heuristique de résolution". *Ann. Telecommunic.,* 32, n. 7-8.