ESTIMATION OF HELICOPTER AND TARGET MOTION FOR THE ADVANCED ATTACK HELICOPTER FIRE CONTROL SYSTEM

R. S. BUCY, S. J. ASSEO, D. A. WEISSENBERGER

The problem of estimating helicopter and target motion for the Advance Attack Helicopter fire control system is formulated using the Kalman-Bucy one-step predictor. Dynamic models for the helicopter and target were developed based on point mass and correlated random accelerations. Observations include own-ship velocity and acceleration, range, line-of-sight (LOS) angles and rates.

The estimation problem is formulated in moving LOS coordinates, where the dynamic models depend on the LOS rate vector, which is assumed noise-free and constant in the sampling interval.

The 9-stage helicopter estimator is omitted and helicopter velocity and acceleration measurements are used directly in the evaluation of the target state estimator performance. A scenario-based simulation program and test data on ground vehicle motion are used in this evaluation. The effect of various error sources, including LOS rate gyro noise, is investigated and the miss distance is computed based on the predicted projectile impact point. It is shown that target state estimates perpendicular to the LOS, which affect the predicted future target position, are quite good as opposed to the target estimates along the LOS.

1. INTRODUCTION

The ability to predict future target positions accurately is of ultimate importance for enhancing the hit probability of unguided weapons. Future target position depends on the weapon time of flight which is strictly a function of target range, as well as on current target state estimates. Hence, the only way to improve the prediction of future target position is to provide accurate estimates of the attacker and target states (position, velocity, acceleration, etc.).

The problem investigated in this paper consist of estimating the states of a ground (or airbone) target and those of the Advance Attack Helicopter, the primary function of which is to engage and kill ground target such as tanks. Kalman-Bucy filtering algorithms are used for estimating these states in rotating line-of-sight (LOS) coordinates, using ownship velocity and acceleration measurements as well as range, LOS angle and LOS rate measurements from a Target Acquisition and Designation Sensor. Time-correlated random acceleration models are used to represent the dynamics of the target and helicopter. Since the state transition matrices and observations associated with this formulation depend on LOS rates ($\underline{\omega}$), $\underline{\omega}$ was treated as constant for state transition equations in the sampling interval to reduce the problem to a linear estimation problem, but also used as a noisy observation of the target velocity!.

This way of treating a variable, which affects both the observation and dynamical model, is novel and quite effective. The effectiveness of this formulation is shown through simulations of helicopter/target engagement scenarios.

2. MODELING THE HELICOPTER AND TARGET IN CONTINUOUS TIME

We represent both the target and helicopter as point masses. The equations of motion are given by

$$\frac{d\underline{x}^*}{dt} = V$$

$$\frac{dv}{dt} = -\rho \; v \; |\underline{v}| + A \qquad (1)^1$$

R.S.Bucy, University of Southern California and Hughes Helicopters, Los Angeles, California.
S.J.Asseo, Hughes Helicopters. Currently with Northrop Aircraft Group.
D.A. Weissenberger, Hughes Helicopters.
- Article rebut el Desembre del 1980.

$$\frac{dA}{dt} = \ell \underline{A} + \underline{u} \tag{1}$$

for the helicopter with $\underline{x}^*$, $\underline{v}$, $\underline{A}$ being three -dimensional position, velocity and pseudo-acceleration[2] vectors in an inertial frame. Similarly, the target motion in inertial space is modeled by

$$\frac{d\underline{x}^T}{dt} = \underline{v}^T$$

$$\frac{d\underline{v}^T}{dt} = -\rho\, \underline{v}^T\, |\underline{v}| + A^T \tag{2}$$

$$\frac{d\underline{A}^T}{dt} = \underline{\lambda}\, \underline{A}^T + \underline{w}$$

where

$\underline{x}^T$, $\underline{v}^T$ and $\underline{A}^T$ = three-dimensional target position, velocity and acceleration vectors in an inertial frame

$\underline{u}$, $\underline{w}$ = white noise processes.

We consider the vector

$$\underline{x}^T - \underline{x}^* = r\, \vec{S}$$

with

$r = |\underline{x}^T - \underline{x}^*|$ the range between the target and the helicopter and denote by $\vec{S}$, $\vec{L}$ and $\vec{M}$ the right-handed moving coordinate system associated with the LOS such that $\vec{S}$ is along the LOS.

The vectors $\vec{S}$, $\vec{L}$ and $\vec{M}$ determine, with their derivatives, an infinitesimal rotation Ω at each instant.

$$\Omega = \begin{pmatrix} 0 & \omega_M & -\omega_L \\ -\omega_M & 0 & \omega_S \\ \omega_L & -\omega_S & 0 \end{pmatrix} \tag{3}$$

through the equations $\dot{\vec{S}} = \underline{\omega} \times \vec{S}$ etc.,

where

$$\underline{\omega} = (\omega_S, \omega_L, \omega_M)^1$$

It will be convenient to consider the joint estimation problem in moving $\vec{S}$, $\vec{L}$, $\vec{M}$ coordinates. We now represent by $\underline{x}$ the 9 x 1 helicopter state vector with components x_1^*, x_2^*, x_3^*, v_1, v_2, v_3, A_1, A_2, A_3 where

$$\underline{x}^* = x_1^* \vec{S} + x_2^* \vec{L} + x_3^* \vec{M}$$

$$\underline{v} = v_1 \vec{S} + v_2 \vec{L} + v_3 \vec{M} \tag{4}$$

$$\underline{A} = A_1 \vec{S} + A_2 \vec{L} + A_3 \vec{M}$$

using the 9 x 1 augmented state vector $\underline{x}$ and writing the helicopter equations of motion in rotating $\vec{S}$, $\vec{L}$, $\vec{M}$ coordinates, yields:

$$\frac{d\underline{x}}{dt} = F\, \underline{x} + G\, \underline{u} \tag{5}$$

with

$$F = \begin{pmatrix} -\Omega & I & 0 \\ 0 & kI-\Omega & I \\ 0 & 0 & \ell I-\Omega \end{pmatrix}$$

$$G = \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix}$$

$$k = -\rho |\underline{v}|$$

The entries in F and G are 3 x 3 matrices.

In LOS coordinates, target position relative to the helicopter is completely defined by specifying rage r (along the LOS $\vec{S}$). Therefore, we only need a seven-dimensional target state vector $\underline{x}^T$ with components $(r, v_1^T, v_2^T, v_3^T, A_1^T, A_2^T, A_3^T)$ to define the relative target motion, where

$$\underline{v}^T = v_1^T \vec{S} + v_2^T \vec{L} + v_3^T \vec{M}$$
$$\tag{6}$$
$$\underline{A}^T = A_1^T \vec{S} + A_2^T \vec{L} + A_3^T \vec{M}$$

The derivative of range is

$$\dot{r} = v_1^T - v_1 \tag{7}$$

where

v_1^T and v_1 = target and helicopter velocities, respectively, along the LOS.

Using Eq. (7) and writing the target equations of motion in rotating $\vec{S}$, $\vec{L}$, $\vec{M}$ coordinates, yields:

$$\frac{d\underline{x}^T}{dt} = F_T\, \underline{x}^T + G_T\, \underline{w} + \xi \tag{8}$$

with

$$F_T = \left|\begin{array}{c|ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & & -\Omega & & & I & \\ \hline 0 & & 0 & & & \lambda I-\Omega & \end{array}\right|$$

$$G_T = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}$$

and

$$e^{-\Omega\Delta} = I - \frac{\sin \omega\Delta}{\omega}\,\Omega + \frac{1 - \cos \omega\Delta}{\omega^2}\,\Omega^2$$

where

$$\omega = |\underline{\omega}| = \sqrt{\omega_S^2 + \omega_L^2 + \omega_M^2}$$

and

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

where

$$\begin{aligned} D_1 &= \frac{1}{k(\ell - k)}\left[k(\ell I - \Omega)^{-1}\left(e^{\ell\Delta}\, e^{-\ell\Delta} - I\right) - (kI - \Omega)^{-1}\left(e^{k\Delta}\, e^{-\Omega\Delta} - I\right)\right] \\ &\quad + \frac{1}{\ell k}\left[\Delta I - \frac{1 - \cos \omega\Delta}{\omega^2}\,\Omega + \frac{\omega\Delta - \sin \omega\Delta}{\omega^2}\,\Omega^2\right] \\ D_2 &= \frac{1}{\ell - k}\left[(\ell I - \Omega)^{-1}\left(e^{\ell\Delta}\, e^{-\ell\Delta} - I\right) - \ell(kI - \Omega)^{-1}\left(e^{k\Delta}\, e^{-\Omega\Delta} - I\right)\right] \\ D_3 &= (\ell I - \Omega)^{-1}\left(e^{(\ell I - \Omega)\Delta} - I\right) \end{aligned} \tag{10}$$

$$\xi = \begin{pmatrix} -v_1 \\ o \\ o \\ o \\ o \\ o \\ o \end{pmatrix}$$

3. DISCRETE TIME MODELS

We suppose that Δ, the time step between measurements, is small enough that $\underline{\omega}$ can be considered constant between measurements, hence, both Eq. (5) and (8) can be integrated to give.

$$x(n\Delta) = \phi\underline{x}((n - 1)\Delta) + D\,\underline{u}$$

with

$$\phi = \begin{bmatrix} e^{-\Omega} & \left(e^{k\Delta} - 1\right)e^{-\frac{\Omega\Delta}{k}} & \frac{1}{\ell - k}\left[\frac{e^{\ell\Delta} - 1}{\ell} - \frac{e^{k\Delta} - 1}{k}\right]e^{-\Omega\Delta} \\ 0 & e^{k\Delta}\, e^{-\Omega\Delta} & \frac{1}{\ell - k}\left(e^{\ell\Delta} - e^{k\Delta}\right)e^{-\Omega\Delta} \\ 0 & 0 & e^{\ell\Delta}\, e^{-\Omega\Delta} \end{bmatrix} \tag{9}$$

Note that we have assumed $\underline{u}$ constant (or sampled) over the interval Δ so that we can write

$$\phi = e^{F\Delta}$$

and

$$D = \int_0^{\Delta} e^{F(\Delta - s)}\, G\,ds$$

We remark that ϕ and D can be replaced by their first two nonvanishing Taylor series terms if Δ is sufficiently small. In a similar manner, the target equation can be integrated to give

$$\underline{x}^T(n\Delta) = \phi^T\, \underline{x}^T\left((n - 1)\Delta\right) + D^T\, \underline{w} \tag{11}$$

where

$$D_1^T = 1/\lambda\ [1,\ 0,\ 0] \left\{ (\lambda I - \Omega)^{-2} (e^{\lambda\Delta} e^{-\Omega\Delta} - I) - (\lambda I - \Omega)^{-1} - 1/2\Delta^2 I + \frac{\omega\Delta - \sin\omega\Delta}{\omega^3}\Omega - \frac{1/2(\Delta\omega)^2 - 1 + \cos\omega\Delta}{\omega^4}\Omega^2 \right\} \quad (13)$$

$$D_2^T = 1/\lambda \left\{ (\lambda I - \Omega)^{-1} (e^{\lambda\Delta} e^{-\Omega\Delta} - I) - I + \frac{1 - \cos\omega\Delta}{\omega^2} \right\} - \frac{\omega\Delta - \sin\omega\Delta}{\omega}$$

$$D_3^T = (\lambda I - \Omega)^{-1} (e^{\lambda\Delta} e^{-\Omega\Delta} - I).$$

These formulae are quite easy to check by differentiation with respect to Δ, and solving the appropriate differential equations.

The reader should note that use of the LOS coordinates has reduced the target state vector dimensionality by two.

with

$$\phi^T = \begin{bmatrix} 1 & \underline{b}(\Delta) & \underline{c}(\Delta) \\ 0 & e^{-\Omega\Delta} & (e^{\lambda\Delta}-1)e^{-\Omega\Delta}/k \\ 0 & 0 & e^{\lambda}\ \ e^{-\Omega\Delta} \end{bmatrix}$$

where

$$b(\Delta) = [1,\ 0,\ 0] \left\{ I\Delta - \frac{(1 - \cos\omega\Delta)}{\omega^2}\Omega + \frac{\omega\Delta - \sin\omega\Delta}{\omega^2}\Omega^2 \right\}$$

$$c(\Delta) = 1/\lambda\ [1,\ 0,\ 0]\ (\lambda I - \Omega)^{-1} \left\{ e^{\lambda\Delta} e^{-\Omega\Delta} - I \right\} - 1/\lambda\ b(\Delta) \quad (12)$$

and

$$\omega = \sqrt{\omega_S^2 + \omega_L^2 + \omega_M^2}$$

Further, D^T is given by:

$$D^T = \begin{bmatrix} D_1^T \\ D_2^T \\ D_3^T \end{bmatrix}$$

4. SENSOR DESCRIPTIONS

The sensor for the helicopter consist of .

		Frequency of Measurement
Ranger:	$\underline{z}_1 = x_1^T + \hat{x}_1 - x_1 + K_1$	$M\Delta$
Doppler:	$\underline{z}_2 = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \begin{pmatrix} K_2 \\ K_3 \\ K_4 \end{pmatrix}$	$S\Delta$
Accelerator:	$\underline{z}_3 = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} - \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} + K \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \begin{pmatrix} K_5 \\ K_6 \\ K_7 \end{pmatrix}$	Δ

(14)

while those of the target consist of

		Frequency of Measurement
Tracker:	$z_1^T = x_1^T + K_1^T$	$j\Delta$
Relative Target Velocity	$\hat{r}\begin{pmatrix} -\omega_M \\ \omega_L \end{pmatrix} = \underline{z}_2^T = \begin{pmatrix} v_2^T \\ v_3^T \end{pmatrix} - \begin{pmatrix} \hat{v}_2 \\ \hat{v}_3 \end{pmatrix} + \begin{pmatrix} K_2^T \\ K_3^T \end{pmatrix}$	Δ

(15)

Relative target velocity is a synthetic measurement made up of the range estimate $\hat{r}$ and the LOS rates ω_L and ω_M perpendicular to the LOS. In Equation (14) and (15), K_i, i=1, 7 and K_i^T, i= 1,2,3 are white measurement --- noise sequences and $\hat{a}$ denotes the estimate of a. In gmeneral, the ranger may be missing and then only $\underline{v}$ and $\underline{A}$ need to be estimated. Notice that for the target dynamics we have used the measurements of $\underline{\omega} = (\underline{\omega}_S, \underline{\omega}_L, \underline{\omega}_M)^1$ - as exact for finding the dynamical matrices ϕ^T and D^T; however, we also view the angular rates multiplied by an estimate of the LOS - range as noisy observations of target velo--city.

The noise vector

$$\begin{pmatrix} K_2^T \\ K_3^T \end{pmatrix}$$

has covariance

$$E(\hat{r}^2)L + E(r-\hat{r})^2 \begin{pmatrix} \omega_M^2 & -\omega_M\omega_L \\ -\omega_M\omega_L & \omega_L^2 \end{pmatrix}$$

where

L = the angular rate noise covariance.

If the angular rates are assumed perfect, -- the order of the tracking filter can be re--duced by eliminating velocities and accelera tions perpendicular to the LOS from the target estimation process. The latter can be - expressed in terms of angular rates, angular accelerations, range estimate $\hat{r}$ and helicopter velocity estimates $\hat{v}_2$, $\hat{v}_3$ as follows:

$$v_2^T = \hat{v}_2 + \hat{r}\omega_M$$

$$v_3^T = \hat{v}_3 - \hat{r}\omega_L$$

$$A_2^T = \hat{r}\dot{\omega}_M + \omega_S\omega_L + 2\hat{\dot{r}}\omega_M$$

$$A_3^T = -\hat{r}\dot{\omega}_L - \omega_S\omega_M - 2\hat{\dot{r}}\omega_L \tag{16}$$

Essentially, only r, $\dot{r}$ and A_1 must be esti--mated and the other states can be found from Eq. (16). The dynamical model used for the estimation process reduces to:

$$\frac{d}{db}\begin{bmatrix} r \\ \dot{r} \\ A_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \omega_L^2 + \omega_M^2 & 0 & 1 \\ 0 & 0 & \lambda \end{bmatrix}\begin{bmatrix} r \\ \dot{r} \\ A_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_1 \end{bmatrix} - \begin{bmatrix} 0 \\ \hat{A}_1 \\ 0 \end{bmatrix} \tag{17}$$

Although this three-state design might have its advantages from a reduced computer size and computation time standpoint, the esti--mates will be sensitive to small amounts of noise in the LOS rate measurements. Therefore, the seven-state tracking filter design will be pursued herein to provide a unila--teral solution to the estimation problem. - The above three-state design was developed by Larry Busick of Hughes Helicopters.

Notice that our problem becomes nicely li--near when $\underline{\omega}$ is know and our way of using $\underline{\omega}$ both as observations and as known for dynamical purposes, seems to be both an effective and a novel method of desing, whether or not it is generally applicable remains to - be seen, although it is quite effective for this problem.

5. FILTER DESIGN

From the description of sensor in the pre--

vious section, the global design is clear - and is given in Figure 1.

Now, from Eq. (14) and (15) H and H_T are as follows:

$$H(i) = \begin{bmatrix} -\overset{m}{\varepsilon}(i) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varepsilon^s(i) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon^s(i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varepsilon^s(i) & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & k & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & k & 0 & 0 & 1 \end{bmatrix}$$

where

$k = - |\hat{\underline{v}}|$

$$H^T(i) = \begin{bmatrix} \varepsilon^n(i) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad (18)$$

where

$\varepsilon^\sigma(i) = 0$

$= 1$

$i \neq 0$ mod

$=$ otherwise

From the particular problem that was presented by the fire control application v_2 and - v_3 could be estimated well enough without -- the construction of a helicopter guidance -- filter by simply using direct sensor outputs Consequently, we will pursue the target filter desing to completion and omit the heli-- copter filter in the remainder of this paper. Of course, in the general case, depending on the noise levels and the sampling interval, it may be necessary to construct the helicopter filter.

The equations for the one-step predictor filter are given by

$$\hat{x}_{n+1|n}^T = \phi^T(n)\hat{x}_{n|n-1}^T + K(n)\left[z_n^T - H^T(n)\hat{x}_{n|n-1}^T\right]$$

$$P_{n+1} = \phi^T(n)\left\{P_n - P_n H^{T'}(n)\left[H^T(n)P_nH^{T'}(n) + R^T(n)\right]^{-1} + H^T(n)P_n\right\}\phi^{T'}(n) + D^T(n)Q^T(n)D^{T'}(n) \qquad (19)^3$$

$$K(n) = \phi^T(n)P_nH^T(n)\left(H^T(n)P_nH^{T'}(n) + R^T(n)\right)^{-1}$$

where

$K(n)$ = the K-B gain matrix.

P_n, $R^T(n)$ and $Q^T(n)$ = the covariances of the error of the state estimate (if the linear approximation is valid) measurement noise and process noise, respectively, defined by: ; see /2/, /3/.

$$P_{n+1} \triangleq Cov\left[\underline{x}_{n+1|n}^{\sim T}\ \underline{x}_{n+1|n}^{\sim T}\right]$$

$$\underline{x}_{n+1|n}^{\sim T} \triangleq \hat{\underline{x}}_{n+1} - \hat{x}_{n+1|n}^{T}$$

and

$$\hat{\underline{x}}_{n+1}^{T} \triangleq E\left[\underline{x}_{n+1}^{T} | \underline{z}_n^{T}, \ldots . \underline{z}_1^{T}, \underline{z}_o^{T}\right]$$

where

$$Cov\ \underline{w}(n), \underline{w}(j) = Q^T(n)\ \delta_{nj}$$

$$Cov\ \underline{k}^T(n), \underline{k}^T(j) = R^T(n)\ \delta_{nj}$$

Note that the dependence of system parameters on time (n) is denoted explicitly since ϕ^T, D^T, sensor noise characteristics and observations vary with $\underline{\omega}$ which dependes on time (n).

The choice of the parameter λ reflects the speed of reaction of the target, whereas the choise of Q^T is based on the acceleration capabilities of the target.

6. SIMULATIONS

The filter described in the last section was analyzed by the means of two simulation programs. The first program produced random accelerations of both helicopter and target models to simulated engagement. Further, closed loop time constants and directions were found for both the helicopter filter and the target filter as a function of the Eigen values and Eigenvectors of the closed loop system. This simulation program was coded by Dave Lucas of Hughes Helicopters. The numerical accuracy for the Eigenvector routines was archieved by the use of Eispack subroutine; see /1/.

The second program used the helicopter sensor outputs directly in the estimation of target states, using a more realistic helicopter/target engagement scenario.

The helicopter was simulated as a rigid body capable of simulataneous rotation and translation relative to earth, while the target was represented as a point mass capable of translation relative to earth. Both vehi---

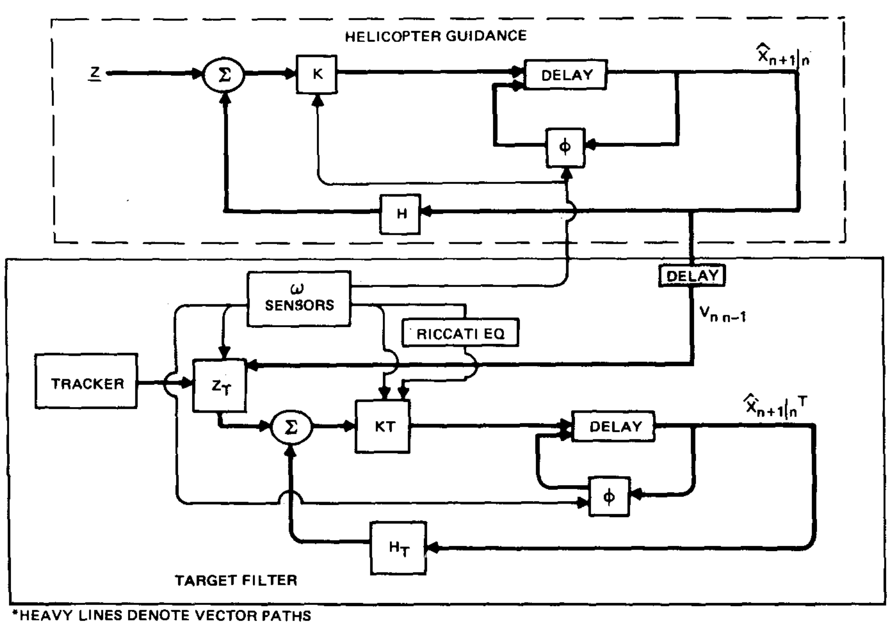

Figure 1. Target State Estimator Block Diagram

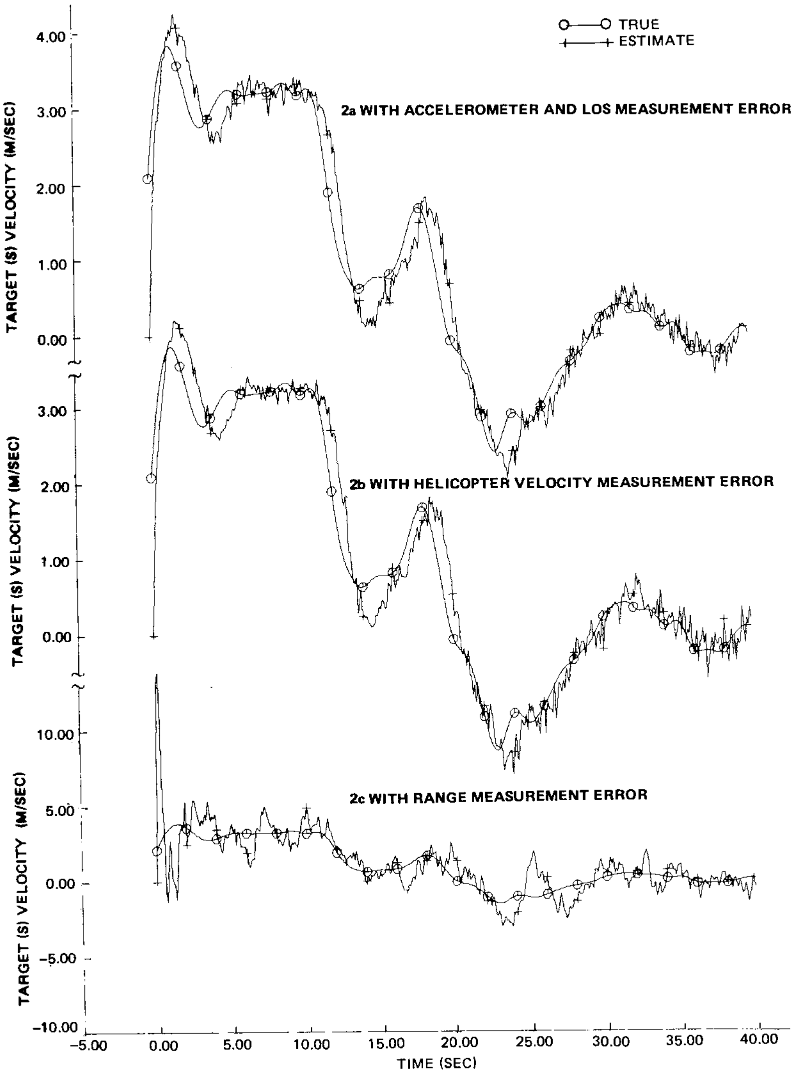

Figure 2. Effect of Sensor Noise on Target Velocity Estimates along the LOS

cles had the freedom to move independently - along a prescribed trajectory or at random, consistent with the respective acceleration capability of each vehicle. Additionally -- the program simulated measurement update --- rates and had the capability of accepting -- real ground target trajectory data. The target estimates perfomance was evaluated using such data.

The effect of sensor noise on target esti--- mates was evaluated by turning each sensor - noise source on and off and examining its -- effect. A typical filter response is shown in Figure 2. From this and similar respon-- ses it became apparent that:

- Range measurement error affects target velocity and acceleration estimates along - the LOS significantly.
- Errors in helicopter acceleration, velocity and LOS directional angle measurements affect the estimates to a lesser extent.
- Bias in helicopter velocity measurement - causes severe deterioration in target estimates.
- LOS rates ($\underline{\omega}$) measurement error affects - the target velocity and acceleration estimates perpendicular to the LOS which are crucial parameters for computing the gun lead angle to predict future target position.

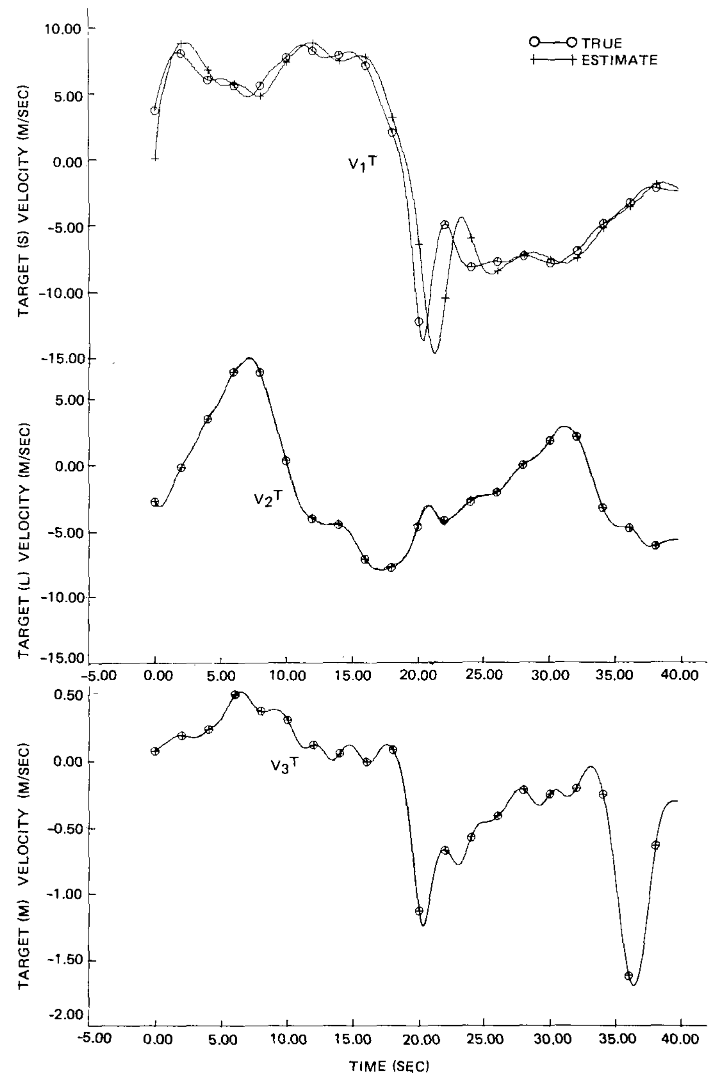

Figure 3. Target Estimator Performance Without Sensor Noise (Sheet 1 of 2)

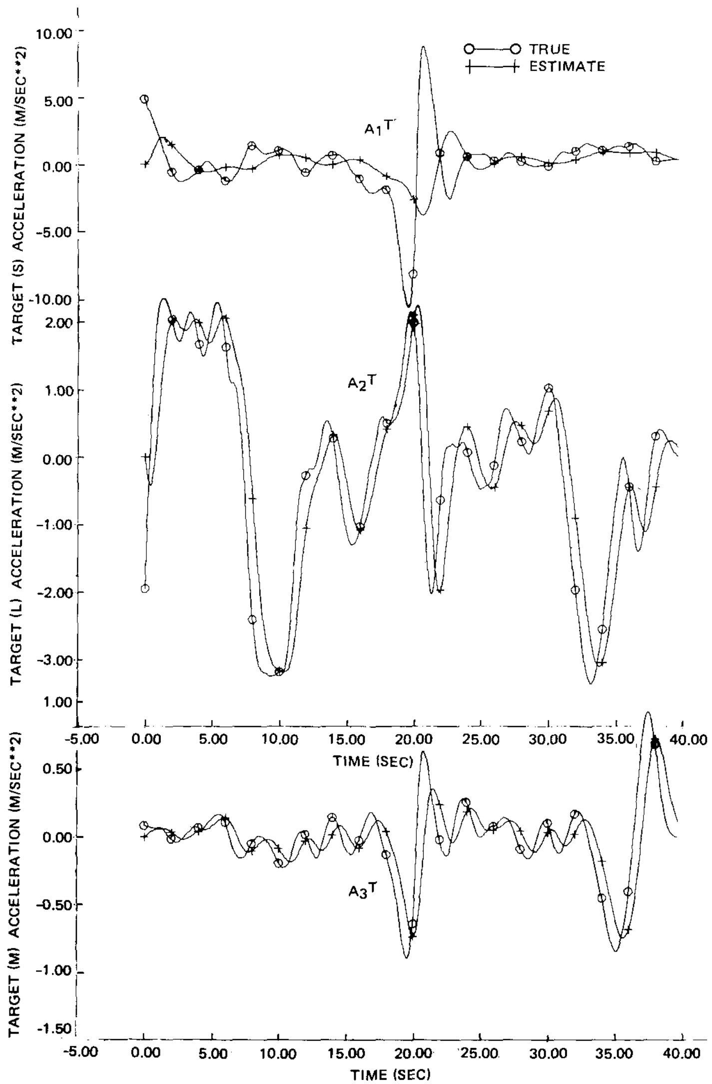

Figure 3. Target Estimator Performance Without Sensor Noise (Sheet 2 of 2)

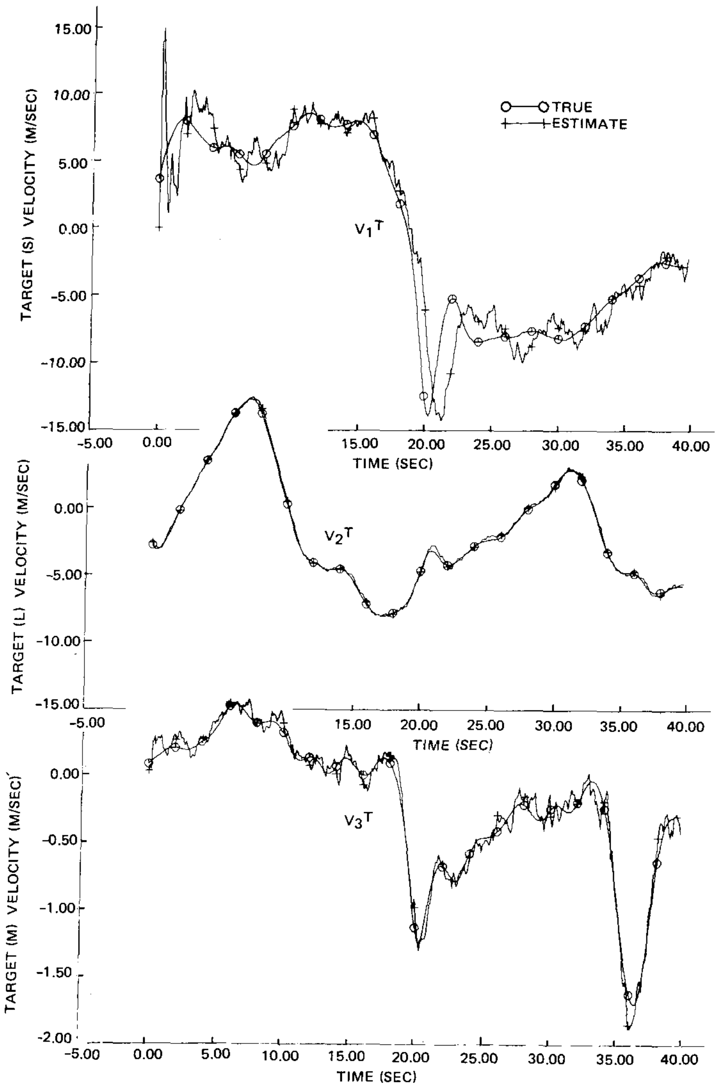

Figure 4. Target Estimator Performance
With Sensor Noise (Sheet 1 of 2)

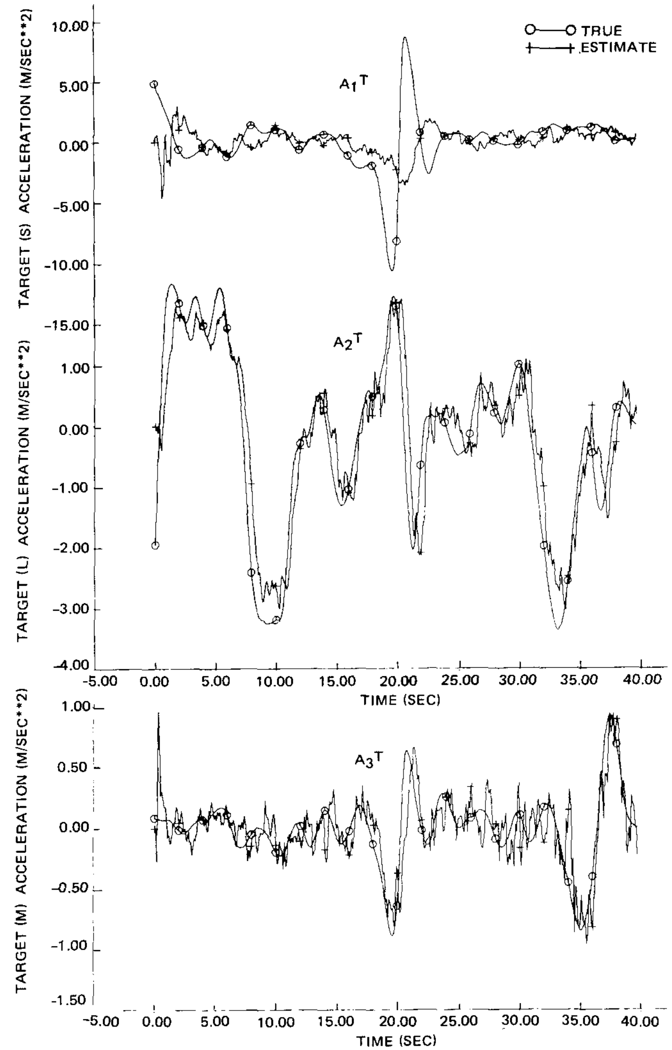

Figure 4. Target Estimator Performance
With Sensor Noise (Sheet 2 of 2)

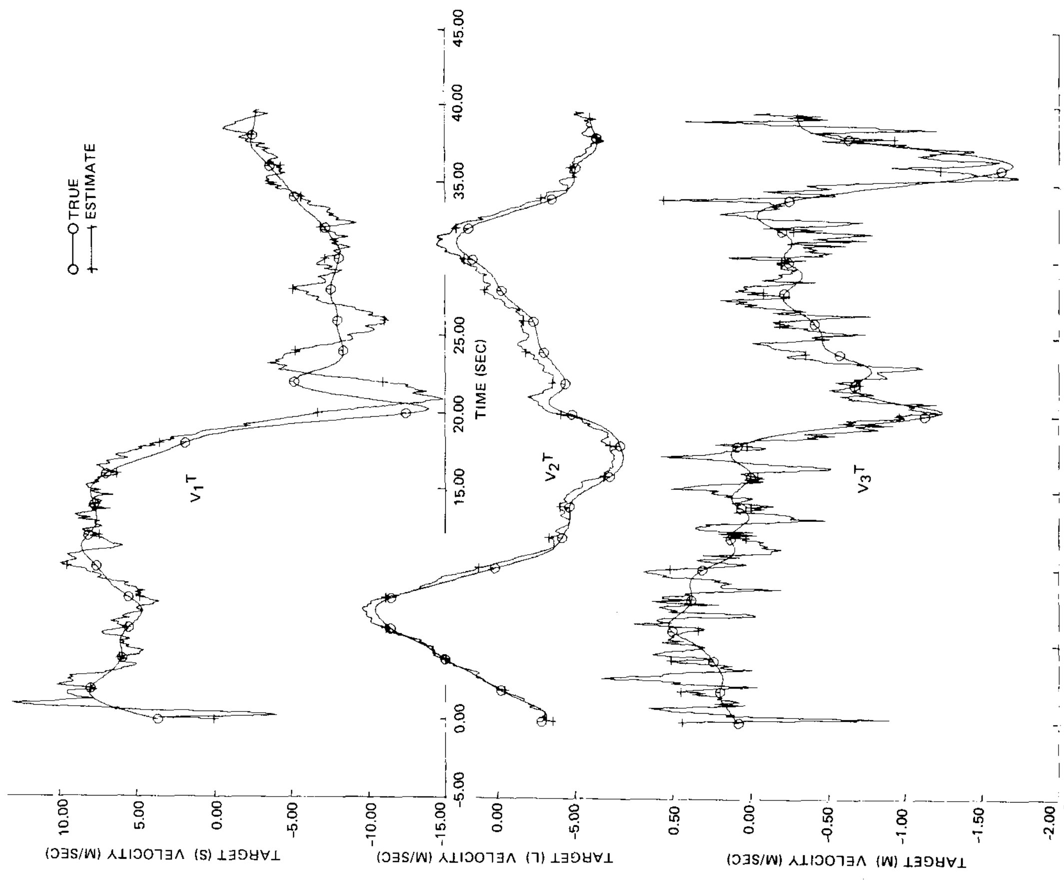

Figure 5. Target Estimator Performance With Sensor Noise and ω Measurement Error (Sheet 1 of 2)

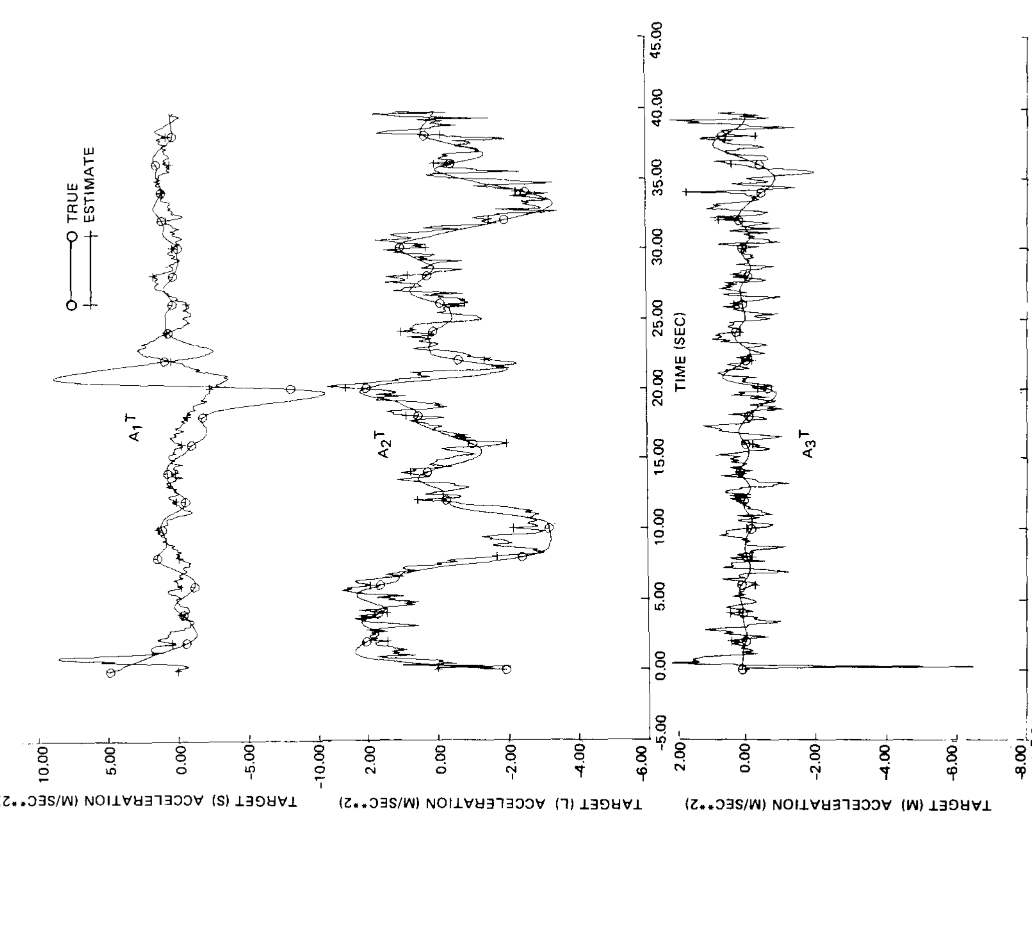

Figure 5. Target Estimator Performance With Sensor Noise and ω Measurement Error (Sheet 2 of 2)

The effect of sensor noise and the overall sensor performance are also depicted in Figures 3, 4 and 5, which show the target filter response without sensor noise, with sensor noise (range, helicopter velocity, acceleration and LOS directional angle measurement error) and no ω-noise and with both sensor noise and ω-noise, respectively. Examination of these figures reveals that the target acceleration estimate along the LOS is the poorest, while target velocity and acceleration estimates perpendicular to the LOS are quite accurate. These results and the tracking filter responses were obtained by using the following sensor noise standard deviations:

Range = 1.67 m

Helicopter Velocity = 0.1 m/sec

Helicopter Acceleration = 0.05 m/sec^2

LOS Directional Angles = 0.2 degrees

LOS Rates = 0.243 millirad/sec random +0.663% Scale Factor Error.

The sensor rates were ten times per second for the tracker and 50 times a second for the angular rates. Larry Busick of Hughes Helicopters took part in the development of the second simulation program.

7. REFERENCES

/1/ EISPACK, A NATS subroutine, U.S. Dept. of Energy, and GARBOW, B.S., BOYLE, J.M. DONGARRA, J.J. and MOLER, C.B., : " Matrix Eigensystem Routines - Eispack Guide Extension", Springer Verlag, New York, 1977.

/2/ KALMAN, E.R. : 'New Methods in Wiener Filtering", Proc. Ist Symp. on Engineering Application of Random for Theory and Prob., Wiley, New York, 1963, pp. 270-389..

/3/ BUCY, R.S., JOSEPH, P.D.: "Filtering for Stochastic Processes with Applications to Guidance", Interscience, New York, 1968.

9. ACKNOWLEDGEMENTS

The design of the target tracker benefitted from inputs from various people, in particular Bill Rea, John Kirby, Larry Busick and Dave Lucas of Hughes Helicopters and Stephen Wolff, AMSAA, Aberdeen Proving Grounds.

This research was sponsored in part by the U.S. Army under Contract Nº DAAJ01-77-C-0064, and the U.S.Air Force under Grant Nº AFOSR-3100.

8. NOTES

[1] Bold-face Latin letters are vector $\underline{v}$ denotes Euclidian norm of vector $\underline{v}$.

[2] For numerical convenience $\underline{A}$ is rotor acceleration plus gravity.

[3] Superscript T denotes target state, and prime denotes matrix transpose.