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ALMOST UNBIASED RATIO AND PRODUCT-TYPE ESTIMATORS IN SYSTEMATIC SAMPLING

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In this paper we have suggested almost unbiased ratio-type and product-type estimators for estimating the population mean $\overline{Y}$ of the study variate y using information on an auxiliary variate x in systematic sampling. The variance expressions of the suggested estimators have been obtained and compared with usual unbiased estimator $\overline{y}^*$, Swain's (1964) ratio estimator $\overline{y}_R^*$ and Shukla's product estimator $\overline{y}_P^*$. It has been shown that the proposed estimators are more efficient than usual unbiased estimator $\overline{y}^*$, ratio estimator $\overline{y}_R^*$ and product estimator $\overline{y}_P^*$. An empirical study is carried out to demonstrate the superiority of the constructed estimators over the estimators $\overline{y}^*$, $\overline{y}_R^*$ and $\overline{y}_P^*$.

Keywords: Almost unbiased ratio and product-type estimators, Auxiliary information, Bias, Variance, Systematic sampling.

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1. INTRODUCTION

Systematic sampling has got the nice feature of selecting the whole sample with just one random start. Apart from its simplicity, which is of considerable importance, this procedure in many situations provides estimators more efficient than simple random sampling and/or stratified random sampling for certain types of population [Cochran (1946, 77), Gautschi (1957), Hajeck (1959)].

Suppose the population consists of N units $U = (U_1, U_2, \ldots, U_N)$ numbered from 1 to N in some order. Unless mention otherwise, we assume $N = nk$, where n and k are positive integers. Thus there will be k samples (clusters) each of size n. We select one sample at random out of k samples and observe the study variate y and auxiliary variate x for each and every unit selected in the sample. Let $(y_{ij}, x_{ij};\quad i = 1,2,\ldots,k;$ $j = 1,2,\ldots,n)$ denote the value of jth unit in the ith sample. The systematic sample means

$$\bar{y}^* = (1/n)\sum_{j=1}^{n} y_{ij}, \quad \bar{x}^* = (1/n)\sum_{j=1}^{n} x_{ij}, \quad (i = 1,2,\ldots,k)$$

are unbiased estimators of the population means $(\overline{Y}, \overline{X})$ of (y,x) respectively. It is assumed that the population mean $\overline{X}$ of the auxiliary variate x is known. Thus the classical ratio and product estimators for $\overline{Y}$ based on a systematic sample $(y_{ij}, x_{ij};$ $i = 1,2,\ldots,k;\quad j = 1,2,\ldots,n)$ of size n, are respectively defined by

$$\bar{y}_R^* = (\bar{y}^*/\bar{x}^*)\overline{X} \tag{1.1}$$

and

$$\bar{y}_P^* = \bar{y}^*(\bar{x}^*/\overline{X}) \tag{1.2}$$

which are respectively due to Swain (1964) and Shukla (1971). The biases and variances of $\bar{y}_R^*$ and $\bar{y}_P^*$ to the first degree of approximation are, respectively, given by

$$B(\bar{y}_R^*) = \frac{(N-1)}{nN}\overline{Y}\{1+(n-1)\rho_x\}(1-k\rho^*)C_x^2 \tag{1.3}$$

$$B(\bar{y}_P^*) = \frac{(N-1)}{nN}\overline{Y}\{1+(n-1)\rho_x\}k\rho^* C_x^2 \tag{1.4}$$

$$\mathrm{Var}(\bar{y}_R^*) = \frac{(N-1)}{nN}\overline{Y}^2\{1+(n-1)\rho_x\}\left[\rho^{*2}C_y^2+(1-2K\rho^*)C_x^2\right] \tag{1.5}$$

$$\mathrm{Var}(\bar{y}_P^*) = \frac{(N-1)}{nN}\overline{Y}^2\{1+(n-1)\rho_x\}\left[\rho^{*2}C_y^2+(1-2K\rho^*)C_x^2\right], \tag{1.6}$$

and the variance of usual unbiased estimator $\bar{y}^*$ is given by

$$\mathrm{Var}(\bar{y}^*) = \frac{(N-1)}{nN}\{1+(n-1)\rho_y\}S_y^2, \tag{1.7}$$

where

$$\rho_x = \frac{E(x_{ij}-\overline{X})(x_{ij}-\overline{X})}{E(x_{ij}-\overline{X})^2}, \qquad \rho_y = \frac{E(y_{ij}-\overline{Y})(y_{ij}-\overline{Y})}{E(y_{ij}-\overline{Y})^2}$$

are intraclass correlation between a pair of units within the systematic sample for the auxiliary variate x and study variate y respectively;

$$\rho = \frac{E(y_{ij}-\overline{Y})(x_{ij}-\overline{X})}{\sqrt{\left\{E(y_{ij}-\overline{Y})^2\, E(x_{ij}-\overline{X})^2\right\}}}$$

is the correlation coefficient between y and x;

$$\rho^* = \left[\{1+(n-1)\rho_y\} / \{1+(n-1)\rho_x\}\right], \quad K = \rho(C_y/C_x)$$

and (C_y, C_x) are the coefficients of variation of the variates (y,x) respectively.

It is obvious from (1.3) and (1.4) that the estimator $\overline{y}^*_R$ and $\overline{y}^*_P$ are biased. In some situations, bias is disadvantegeous. To keep this in view Kushwaha and Singh (1989) suggested a class of almost unbiased ratio and product-type estimators for population mean $\overline{Y}$ using Jack-knife technique introduced by Quenouille (1956). However, it has been observed that their estimators attained the minimum variance equals to the approximate variance of usual linear regression estimator in systematic sampling, for the well known optimal choice $K^* = \rho^* K$. Later Banarasi et al (1993) suggested a ratio, product and difference estimator in systematic sampling. Banarasi *et al* (1993) estimator attains the minimum variance equals to the approximate variance of usual linear regression estimator for the known value of K. However the optimum estimator in the class of estimators suggested by Banarasi *et al* (1993) is biased. In this paper an effort has been made to propose almost unbiased ratio and product-type estimators which depend only on the well known optimum choice $K^* = \rho^* K$. The value of K can be made known quite accurately either due to past experience or by pilot sample surveys. Various authors including Murthy (1967, p. 325), Reddy (1978) and Srivenkataramana and Tracy (1983) have advocated about the assessment of the value of K^*.

2. THE CLASS OF RATIO-TYPE ESTIMATORS

Let the correlation between y and x be positive. Suppose $d_1 = \overline{y}^*$, $d_2 = \overline{y}^*(\overline{X}/\overline{x})$ and $d_3 = \overline{y}^*(\overline{X}/\overline{x}^*)$ such that $d_i \in D \quad (i = 1,2,3)$, where D denotes the set of all possible ratio-type estimators for estimating population mean $\overline{Y}$. By definition, the class D will consist of all d_r of the form

$$d_r = \sum_{i=1}^{3} w_i d_i \in D \tag{2.1}$$

where

$$\sum_{i=1}^{3} w_i = 1 \quad \text{for} \quad w_i \in R \tag{2.2}$$

and w_i $(i = 1, 2, 3)$ denotes the constants used for reducing the bias in the class of estimators and R stands for the set of real numbers. Such procedure of estimation of mean has also been discussed by Singh and Singh (1993).

3. VARIANCE

To obtain the variance of d_r to the first degree of approximation, we write

$$e_0 = (\overline{y}^* - \overline{Y})/\overline{Y}, \quad e_1 = (\overline{x}^* - \overline{X})/\overline{X}$$

such that

$$\mathrm{E}(e_0) = \mathrm{E}(e_1) = 0$$

and

$$\begin{aligned}
\mathrm{E}(e_0^2) &= \frac{(N-1)}{nN}\{1 + (n-1)\rho_y\}C_y^2 \\
\mathrm{E}(e_1^2) &= \frac{(N-1)}{nN}\{1 + (n-1)\rho_x\}C_x^2 \\
\mathrm{E}(e_0 e_1) &= \frac{(N-1)}{nN}\{1 + (n-1)\rho_y\}^{1/2}\{1 + (n-1)\rho_x\}^{1/2}\rho C_y C_x.
\end{aligned}$$

Expressing (2.1) in terms of e's with (2.2), we have

$$d_r = \overline{Y} + \overline{Y}\left[e_0 - (w_2 + 2w_3)e_1 + O(e^2)\right]. \tag{3.1}$$

Let us choose

$$w_2 + 2w_3 = w \quad \text{(say, another constant)} \tag{3.2}$$

Then, to the first degree of approximation, the variance of d_r is given by

$$\mathrm{Var}(d_r) = \frac{(N-1)}{nN}\overline{Y}^2\{1 + (n-1)\rho_x\}\left[\rho^{*2}C_y^2 + w(w - 2\rho^* K)C_x^2\right] \tag{3.3}$$

which is minimized for

$$w = \rho^* K = K^* \quad \text{(say)} \tag{3.4}$$

Substitution of (3.4) in (3.3) yields the minimum variance of d_r as

$$\text{min. Var}(d_r) = \frac{(N-1)}{nN}\{1+(n-1)\rho_y\}(1-\rho^2)S_y^2 \tag{3.5}$$

where $S_y^2 = \dfrac{N}{(N-1)}\mathrm{E}(y_{ij}-\overline{Y})^2$.

We have thus proved the following theorem.

Theorem 3.1. *Up to terms of order n^{-1},*

$$Var(d_r) \geq \frac{(N-1=}{nN}\{1+(n-1)\rho_y\}(1-\rho^2)S_y^2$$

with equality holding if

$$w = \rho^* K = K^*.$$

4. BIAS REDUCTION OF ORDER $O(n^{-1})$

Equation (3.5) shows that the class of estimators attain the minimum variance equal to that of the usual linear regression estimator $\overline{y}_{1r}$ in systematic sampling, defined as

$$\overline{y}_{1r} = \overline{y}^* + \hat{\beta}^*(\overline{X} - \overline{x}^*) \tag{4.1}$$

where $\hat{\beta}^*$ is the systematic sample regression coefficient of y on x.

From (3.2) and (3.4), we have

$$w_2 + 2w_3 = \rho^* K \tag{4.2}$$

We note from (2.2) and (4.2) that there are three unknown quantities to be determined from only two equations. It is, therefore, not possible to obtain unique values for the constants w_i's, $(i = 1,2,3)$ to be used for bias reduction. To get the unique values for these constants w_i's, $(i = 1,2,3)$, we shall impose the additional linear restriction as

$$\sum_{i=1}^{3} B(d_i) = 0 \tag{4.3}$$

where $B(d_i)$ stands for the bias in the $i-$th $(i = 1,2,3)$ estimator of the population mean $\overline{Y}$.

Equations (2.2), (4.2) and (4.3) may be expressed as

$$\text{(4.4)} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ B(d_1) & B(d_2) & B(d_3) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \rho^* K \\ 0 \end{bmatrix}$$

It is well known in systematic sampling that

$$\text{(4.5)} \qquad B(d_1) = B(\overline{y}^*) = 0$$

and to the first degree of approximation

$$\text{(4.6)} \qquad B(d_3) = \frac{(N-1)}{nN}\overline{Y}\,\{1+(n-1)\rho_x\}\,(3-2\rho^* K).$$

The bias of d_2 is given at (1.3). Thus using (1.3), (4.5) and (4.6) in (4.4), we get the values of w_1, w_2 and w_3 as

$$\text{(4.7)} \qquad \left.\begin{aligned} w_1 &= (1-k\rho^*)^2 \\ w_2 &= (3-2k\rho^*)K\rho^* \\ w_3 &= (K\rho^*-1)K\rho^* \end{aligned}\right\}$$

Substitution of (4.7) in (2.1) yields an almost unbiased ratio-type estimator for $\overline{Y}$ as

(4.8)

$$d_{ru} = \left[(1-K\rho^*)^2\overline{y}^* + (3-2K\rho^*)K\rho^*\overline{y}^*(\overline{X}/\overline{x}^*) - (1-K\rho^*)K\rho^*\overline{y}^*(\overline{X}/\overline{x}^*)^2\right]$$

with the variance

$$\text{(4.9)} \qquad \mathrm{Var}(d_{ru}) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_y\}\,(1-\rho^2)S_y^2.$$

We have thus proved the following theorem.

Theorem 4.1. *The estimator d_{ru} at (4.8) is an «optimum almost unbiased ratio-type» in the class of ratio-type estimators d_r at (2.1), with the variance given at (4.9).*

It is to be noted that the estimator d_{ru} at (4.8) is unbiased upto terms of order n^{-1}. Same process may be repeated by considering $B(d_i)$, $(i = 1,2,3)$ to terms of order $O(n^{-2})$, to get the unbiased ratio-type estimator to terms of order $O(n^{-2})$ and so on.

In many situations of practical importance $\rho_y \simeq \rho_x$ and known, for instance, see Murthy (1967) and Srivenkataramana and Tracy (1983). Thus $\rho^* \to 1$ and $K \to K$ and hence the values of w_i's, $(i = 1,2,3)$ are given by

$$\left.\begin{aligned} w_1 &= (1-K)^2 \\ w_2 &= (3-2K)K \\ w_3 &= K(K-1) \end{aligned}\right\} \tag{4.10}$$

Putting (4.10) in (2.1) we get the almost unbiased ratio-type estimator for $\overline{Y}$ as

(4.11)

$$d^*_{ru} = \left[(1-K)^2\overline{y}^* + K(3-2K)\overline{y}^*(\overline{X}/\overline{x}^*) + K(K-1)\overline{y}^*(\overline{X}/\overline{x}^*)^2\right]$$

with the variance

$$\mathrm{Var}(d^*_{ru}) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_y\}(1-\rho^2)S_y^2. \tag{4.12}$$

It is further remarked that the estimator d^*_{ru} in (4.11) can also be obtained from the estimator d_{ru} with $\rho^* \simeq 1$. The value of K can easily be guessed quite accurately either through pilot survey or experienced gathered in due course of time, for instance, see Murthy (1967, p. 325) and Reddy (1978).

From (1.5), (1.7) and (4.9) we have

$$\mathrm{Var}(\overline{y}^*) - \mathrm{Var}(d_{ru}) = \frac{(N-1)}{nN}\{1+(n-1)\rho_y\}\rho^2 S_y^2 > 0 \tag{4.13}$$

and

$$\begin{aligned} \mathrm{Var}(\overline{y}^*_R) - \mathrm{Var}(d_{ru}) &= \frac{(N-1)}{nN}\overline{Y}^2\{1+(n-1)\rho_x\}C_x^2(1-K\rho^*)^2. \\ &> 0 \quad \text{unless } K\rho^* = 1. \end{aligned} \tag{4.14}$$

We have thus established the following theorems.

Theorem 4.2. *The inequality*

$$Var(d_{ru}) < Var(\overline{y}^*)$$

always holds good.

Theorem 4.3. *The inequality*

$$Var(d_{ru}) < Var(\overline{y}_R^*)$$

is always true except when $K\rho^* = 1$.

Remark 4.1. It is customary in systematic sampling to arrange the population units such that ρ_y, ρ_x are small, preferably negative, since positive ρ_y, ρ_x inflate sampling variance. Thus it may be possible to assess ρ_y, ρ_x quite accurately for the arrangement used. This together with assessed $K = \rho(C_y/C_x)$ leads to the optimal choice of K^*.
□

5. A CLASS OF PRODUCT-TYPE ESTIMATORS

Suppose $d_1^* = \overline{y}^*$, $d_2^* = \overline{y}^*(\overline{x}^*/\overline{X})$ and $d_3^* = \overline{y}^*(\overline{x}^*/\overline{X})^2$ such that $d_i^* \in D^*$ for $i = 1, 2, 3$, where D^* denotes the sets of all possible product-type estimators for estimating the population mean $\overline{Y}$. By definition, the set D^* will consist of all d_p^* of the form

$$d_p^* = \sum_{i=1}^{3} w_i^* \, d_i^* \in D^* \tag{5.1}$$

for

$$\sum_{i=1}^{3} w_i^* = 1 \quad \text{and} \quad d_i^* \in R \tag{5.2}$$

where w_i's, $(i = 1, 2, 3)$ denote the constants used for bias reduction.

As in section 3, the following theorem can easily be proved.

Theorem 5.1. *Up to terms of order* n^{-1},

$$Var(d_p^*) \geqslant \frac{(N-1)}{nN} \{1 + (n-1)\rho_y\} (1 - \rho^2) S_y^2$$

with equality holding if

$$w = -\rho^* K = -K^*,$$

where $w^* = (w_2^* + 2\,w_3^*)$.

Proceeding exactly in the same way as in section 2,3 and 4, we get the values of w_i's, $(i=1,2,3)$ as

$$\left.\begin{aligned} w_1^* &= [1+\rho^* K(1+\rho^* K)] \\ w_2^* &= -\rho^* K(1+2\rho^* K) \\ w_3^* &= \rho^{*2} K^2. \end{aligned}\right\} \tag{5.3}$$

Use of these w_i's, $(i=1,2,3)$ removes the bias of d_p^* upto terms of order n^{-1} at (5.1). Thus the substitution of w_i's, $(i=1,2,3)$ in (5.1) yields an almost unbiased product-type estimator

(5.4)
$$d_{pu}^* = \left[\{1+\rho^* K(1+\rho^* K)\}\overline{y}^* - \rho^* K(1+2K\rho^*)\overline{y}^*(\overline{x}^*/\overline{X}) + \rho^{*2}K^2\overline{y}^*(\overline{x}^*/\overline{X})^2\right]$$

with the variance

$$Var(d_{pu}^*) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_y\}(1-\rho^2)S_y^2. \tag{5.5}$$

In case $\rho_y \simeq \rho_x \Rightarrow \rho^* = 1$, the expressions in (5.3) reduce to:

$$\left.\begin{aligned} w_1^* &= [1+K(1+K)] \\ w_2^* &= -K(1+2K) \\ w_3^* &= K^2 \end{aligned}\right\} \tag{5.6}$$

and hence the estimator d_{pu}^* at (5.4) takes the form

$$d_{pu}^{**} = \left[\{1+K(1+K)\}\overline{y}^* - K(1+2K)\overline{y}^*(\overline{x}^*/\overline{X}) + K^2\overline{y}^*(\overline{x}^*/\overline{X})^2\right] \tag{5.7}$$

with the variance given by

$$Var(d_{pu}^{**}) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_y\}(1-\rho^2)S_y^2. \tag{5.8}$$

We have thus proved the following theorem.

Theorem 5.2. *The estimator d_{pu}^* at (5.4) is an «optimum almost unbiased product-type» in the class of product-type estimators d_p^* at (5.1), with the variance given at (5.5).*

From (1.6), (1.7) and (5.5) we have

$$Var(\overline{y}^{*}) - Var(d^{*}_{pu}) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_y\}\,\rho^2 S_y^2 \;>\; 0, \tag{5.9}$$

and

$$Var(\overline{y}^{*}_{p}) - Var(d^{*}_{pu}) = \frac{(N-1)}{Nn}\{1+(n-1)\rho_x\}\,(1+K\rho^{*})^2 \tag{5.10}$$

$$> O \quad \text{provided}\; K\rho^{*} \neq -1.$$

We have thus established the following theorems.

Theorem 5.3. *The inequality*

$$Var(d^{*}_{pu}) \;<\; Var(\overline{y}^{*})$$

is always true.

Theorem 5.4. *The inequality*

$$Var(d^{*}_{pu}) \;<\; Var(\overline{y}^{*}_{p})$$

always holds good except when $K\rho^{*} = -1$.

6. EMPIRICAL STUDY

In this section, the relative efficiencies of almost unbiased ratio-type estimator d and product-type estimator d^{*}_{pu} have been evaluated with the help of live data of Population I and Population II respectively.

POPULATION I

To see the effect of different sample sizes on the approximate relative variances of $\overline{y}^{*}$, $\overline{y}^{*}_{R}$ and d_{ru}, the data on volume of timber of 176 forest strips given in Murthy [1967, p. 131-132] have been considered. The value of intraclass correlation coefficient $\rho_y \simeq \rho_x = \rho_w$ (say) have been given by Murthy [1967, p. 149] and Kushwaha and Singh (1989) for different systematic samples of sizes 4, 8, 16 and 22 strips by enumerating all possible systematic samples after arranging the data in ascending order of strip length. For the systematic sampling to be efficient, the units within the same systematic sample should be as heterogenous as possible with respect to the characteristic under consideration. As the volume (y) of timber is expected to be related to the strip length (x), the arrangement according to this is likely to be approximately similar to the arrangement according to the volume of timber, the study variable y.

The intraclass correlations ρ_w were computed by the formula

$$\rho_w = 1 - \frac{n}{(n-1)}(\rho_w^2/\rho^2), \tag{6.1}$$

where ρ^2 and ρ_w^2 are the population and within sample variances respectively, given as

$$\sigma^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{X})^2,$$

$$\sigma_w^2 = \frac{1}{k}\sum_{i=1}^{k}\sigma_{wi}^2 = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{x}_i)^2.$$

The intraclass correlation ρ_w generally varies with the sample size and arrangement of units in the population. Summarized data are as follows:

$$N = 176, \quad Y = 282.6136, \quad X = 6.9943, \quad C = 0.3036,$$

$$C = 0.1791, \quad S = 309.8317, \quad r = 0.6722, \quad K = 0.8752.$$

The values of constants w_i's $(i = 1,2,3)$, the relative variances of $\overline{y}^*$, $\overline{y}_R^*$ and d_{ru}, and the percent relative efficiencies (PRE's) of $\overline{y}_R^*$ and d_{ru} with respect to $\overline{y}^*$ have been computed for different values of n and displayed in Table 6.1.

Table 6.1. *Showing the relative variances and PRE's of different estimators of* $\overline{Y}$

	Sample size *n*			
	4	8	16	22
ρ_w	-0.1510	-0.1106	-0.0522	-0.0435
w_1	0.0156	0.0156	0.0156	0.0156
w_2	1.0937	1.0937	1.0937	1.0937
w_3	-0.1092	-0.1092	-0.1092	-0.1092
$RV(\overline{y}^*)$	$4.1281 \cdot 10^{-2}$	$0.8521 \cdot 10^{-2}$	$0.4094 \cdot 10^{-2}$	$0.1187 \cdot 10^{-2}$
$RV(\overline{y}_R^*)$	$2.3007 \cdot 10^{-2}$	$0.4749 \cdot 10^{-2}$	$0.2282 \cdot 10^{-2}$	$0.0662 \cdot 10^{-2}$
$Rv(d_{ru})$	$2.2628 \cdot 10^{-2}$	$0.4671 \cdot 10^{-2}$	$0.2244 \cdot 10^{-2}$	$0.0651 \cdot 10^{-2}$
$RE(\overline{y}_R^*, \overline{y}^*)$	179.43	179.43	179.43	179.43
$RE(d_{ru}, \overline{y}^*)$	182.45	182.45	182.45	182.45

Table 6.1 exhibits that the performance of the proposed almost unbiased ratio-type estimator d_{ru} is better than the usual unbiased estimator $\bar{y}^*$ and Swain's (1964) ratio estimator $\bar{y}_R^*$.

POPULATION II

To see the effect of different sample sizes on the approximate relative variances of $\bar{y}^*$, $\bar{y}_P^*$ and d_{pu}^*, the live data of population of size $N = 16$, on per capita consumption (y) and deflated prices (x) of veal reported in Maddala [1977, p. 98] have been considered. The value of intraclass correlation coefficient $\rho_y \simeq \rho_x = \rho_w$ (say) have been computed for different systematic samples of sizes 2, 4 and 8 by enumerating all possible systematic samples after arranging the data in ascending order of consumption. Summarized data are as follows:

$$N = 16, \quad Y = 7.6375, \quad X = 75.4313, \quad C = 0.0519,$$
$$C = 0.0097, \quad S = -8.8319, \quad r = -0.6823, \quad K = -1.5805.$$

The values of constants w_i's, $(i = 1,2,3)$, the relative variances of $\bar{y}^*$, $\bar{y}_P^*$ and d_{pu}^*, and the percent relative efficiencies (PRE's) of $\bar{y}_P^*$ and d_{pu}^* with respect to $\bar{y}^*$ have been computed for various values of n and shown in Table 6.2.

Table 6.2. *Showing the relative variances and percent relative efficiencies (PRE's) of different estimators of* $\bar{Y}$

	Sample size n		
	2	4	8
ρ_w	-0.7220	-0.2744	-0.0932
w_1^*	1.9175	1.9175	1.9175
w_2^*	-3.4155	-3.4155	-3.4155
w_3^*	2.4980	2.4980	2.4980
$RV(\bar{y}^*)$	$6.7632 \cdot 10^{-3}$	$4.3012 \cdot 10^{-3}$	$2.1141 \cdot 10^{-3}$
$RV(\bar{y}_P^*)$	$4.0316 \cdot 10^{-3}$	$2.5640 \cdot 10^{-3}$	$1.2602 \cdot 10^{-3}$
$RV(d_{pu}^*)$	$3.6147 \cdot 10^{-3}$	$2.2989 \cdot 10^{-3}$	$1.1299 \cdot 10^{-3}$
$RE(\bar{y}_P^*, \bar{y}^*)$	167.76	167.76	167.76
$RE(d_{pu}^*, \bar{y}^*)$	187.10	187.10	187.10

Table 6.2 clearly indicates that the proposed almost unbiased product-type estimator d_{pu}^* is more efficient than usual unbiased estimator $\bar{y}^*$ and Shukla's (1971) product estimator $\bar{y}_P^*$.

7. CONCLUSIONS

It is observed from (1.3) and (1.4) that both the estimators $\overline{y}_R^*$ and $\overline{y}_P^*$ suggested by Swain (1964) and Shukla (1971) respectively, are biased, which is a drawback in some practical situations while the suggested estimators d_{ru} and d_{pu}^* are almost unbiased. We note from (4.13) and (5.9) that the estimators d_{ru} and d_{pu}^* are always better than systematic sample mean estimator $\overline{y}^*$. It is further, observed from (4.14) and (5.10) that the suggested estimator d_{ru} (d_{pu}^*) is always better than $\overline{y}_R^*$ ($\overline{y}_P^*$) except when $K\rho^* = 1(K\rho^* = -1)$, the case where both the estimators $d_{ru}(d_{pu}^*)$ and $\overline{y}_R^*$ ($\overline{y}_P^*$) are equally efficient [see Tables 6.1 and 6.2].

Thus we conclude that the suggested estimator d_{ru} (d_{pu}^*) is superior to $\overline{y}_R^*$ ($\overline{y}_P^*$) according to both the criterion unbiasedness as well as variance.

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